

# Radiation energy transfer and maximum conversion efficiency

Kuan Chen<sup>a,\*</sup>, Wongee Chun<sup>b,1</sup>

<sup>a</sup> Department of Mechanical Engineering, University of Utah, SLC, UT 84112, USA

<sup>b</sup> Department of Nuclear and Energy Engineering, Cheju National University, Cheju 690-756, Republic of Korea

## ARTICLE INFO

### Article history:

Received 29 July 2008

Received in revised form 10 November 2008

Accepted 14 November 2008

Available online 25 December 2008

### Keywords:

Blackbody radiation  
Conversion efficiency  
Photon gas

## ABSTRACT

Radiation energy transfer is modeled as the enthalpy flux of photons across the boundary of a thermodynamic system. It is proved that this energy transfer process can be treated as heat transfer. Compression work must be applied to the system to push the photons out. The energy transfer rate and maximum conversion efficiency computed from the model are identical to those determined from the Stefan–Boltzmann law and the Carnot efficiency for blackbody radiation.

© 2008 Elsevier Ltd. All rights reserved.

## 1. Introduction

Of the three heat transfer modes, radiation is probably the energy transfer process most important to all life forms on earth. It is the only way energy can be transferred from the sun to the earth. Recent soaring energy prices and increased concerns about global warming and other environmental issues have attracted a lot of attention and interest on solar radiation. A vast amount of work on solar energy utilization and conversion has been published in the past.

Various analyses based on classical or statistical thermodynamics or quantum mechanics (e.g. [1–13]) have been carried out to determine the maximum conversion efficiencies of solar and other radiation sources. In these publications either the Carnot efficiency was applied to a cyclic device operating between two thermal reservoirs, or the change in exergy of a photon gas was calculated to determine the maximum production of high-grade energy (e.g. electricity or mechanical work) in a process.

The mechanism of radiation transfer is the propagation of electromagnetic waves [14], which can be modeled as the flow of a photon gas. Thermodynamic properties of photon gas have been determined and studied in detail. The total number of photons, internal energy, entropy, and pressure of a photon gas at temperature  $T$  are [15,16]

$$N = aT^3V \quad (1)$$

$$U = bT^4V \quad (2)$$

$$S = \frac{4}{3}bT^3V \quad (3)$$

$$P = \frac{1}{3}bT^4 \quad (4)$$

where  $a$  and  $b$  are constants (functions of the speed of light, the Planck constant, and the Boltzmann constant), and  $V$  the volume of the photon gas. All electromagnetic waves travel at the speed of light  $c$ . The flux of internal energy of a photon gas across a surface can be calculated from [16]

$$J_u = U \frac{c}{4V} = \sigma T^4 \quad (5)$$

where  $\sigma$  is the Stefan–Boltzmann constant. The entropy flux of a photon gas also can be calculated from its specific entropy,  $S/V$ , and the speed of light

$$J_s = \frac{4}{3}\sigma T^3 \quad (6)$$

Because the internal energy flux of a photon gas is not equal to the product of its entropy flux and temperature, some researchers pointed out that electromagnetic wave propagation (or the flow of a photon gas) should not be modeled as heat transfer. Consequently the Carnot corollaries are not applicable for cycles receiving and/or rejecting radiation in their opinions.

## 2. Previous investigations on maximum conversion efficiency of radiation energy

Computing the exergy change of a fixed quantity of photon gas of constant volume (no boundary work [17] can be done on or by

\* Corresponding author. Tel.: +1 801 5814150; fax: +1 801 5859826.  
E-mail addresses: [chen@mech.utah.edu](mailto:chen@mech.utah.edu) (K. Chen), [wgchun@cheju.ac.kr](mailto:wgchun@cheju.ac.kr) (W. Chun).

<sup>1</sup> Tel./fax: +82 64 7579276.

the photon gas in this case), Jeter [2] proved that the maximum work output of a Carnot heat engine that extracts heat from the photon gas and rejects heat to a thermal reservoir at  $T_o$  (the final temperature of the photon gas) is

$$W = U_s \left[ 1 - \frac{4}{3} \frac{T_o}{T_s} + \frac{1}{3} \left( \frac{T_o}{T_s} \right)^4 \right] \quad (7)$$

where  $U_s$  and  $T_s$  are the initial internal energy and temperature of the photon gas, respectively. This is the same result as the maximum work potential found in Gyftopoulos and Beretta's book [7], Landsberg and Tonge's paper [1], and other early publications.

Jeter then extended the result given by Eq. (7) to steady flow of radiant energy in an open-end, piston–cylinder device, and found the maximum conversion efficiency for steady radiant transfer is the Carnot efficiency

$$\eta = \frac{\text{(internal energy of photon gas entering the cylinder)}}{\text{(network output of the cylinder)}} - \text{Carnot engine combination} = 1 - (T_o/T_s) \quad (8)$$

A solar conversion system consisting of an ideal concentrator, a radiant cavity, and a Carnot heat engine was also investigated in Jeter's paper.

Other radiation converter designs for which radiation was not treated as heat transfer have also been studied (e.g. [3,5,7]). Depicted in Fig. 1 is a photo-thermal conversion system discussed in [2–4,8,11,18]. Since radiation was not modeled as heat transfer, a radiation absorber (or several absorbers) was installed between the radiation source and a Carnot heat engine in Fig. 1. The radiation absorbed by the absorber(s) is converted into heat and used as the heat input for the Carnot engine. The maximum conversion efficiency for this system with a single absorber was found to be

$$\eta = \left( 1 - \frac{T_a^4}{T_H^4} \right) \left( 1 - \frac{T_L}{T_a} \right) \quad (9)$$

and the absorber temperature that maximizes the efficiency is the positive real root of the equation

$$4T_a^5 - 3T_L T_a^4 - T_L T_H^4 = 0 \quad (10)$$

The absorber in Fig. 1 must be heated up and cooled down during startup and shut down if its initial and final temperatures are not  $T_a$ . In addition, it is clear some entropy is generated when photons emitted by the blackbody source at  $T_H$  are absorbed by the absorber at a lower temperature  $T_a$  without producing any work. Badescu [11] proved that if the irreversibilities associated with filling a solar energy converter with and emptying it of radiation were taken into account, the maximum efficiency of the radiation converter was the Landsberg–Petela–Press efficiency

$$\eta = 1 - \frac{4}{3} \frac{T_L}{T_H} + \frac{1}{3} \left( \frac{T_L}{T_H} \right)^4 \quad (11)$$

With all processes treated as reversible, the conversion efficiency becomes the Carnot efficiency. Jeter pointed out in [4] that there may be radiation converter designs other than the one depicted in Fig. 1 that can totally eliminate the entropy generation

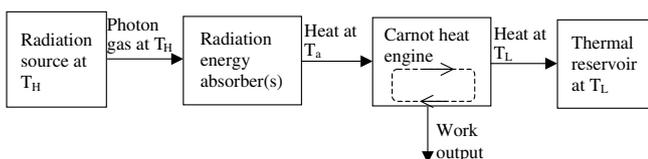


Fig. 1. A photo-thermal converter for converting radiation energy into work.

during radiation energy conversion, thus yielding the Carnot efficiency. Boehm [6] pointed out that back radiation was assumed zero in Jeter's derivation of the Carnot efficiency. Although Jeter's analysis is not in violation of any physics laws, innovative designs (perhaps solid-state devices) other than conventional heat engines that involve a working fluid are probably needed to achieve the Carnot efficiency.

### 3. The present model for radiation energy transfer and conversion

A model is proposed to demonstrate that electromagnetic waves (or photons) absorbed or emitted by a thermodynamic system or matter can be treated as heat transfer. The resultant energy flux and maximum conversion efficiency are identical to those determined from the Stefan–Boltzmann law and the Carnot efficiency.

We first consider the exchange of radiation energy between two blackbodies ( $BB_H$  and  $BB_L$ ) at different temperatures  $T_H$  and  $T_L$ , as shown in Fig. 2. Since a photon gas has internal energy as well as pressure, the total energy flow across the surface area  $A_H$  of the high-temperature blackbody  $BB_H$  should be the flow of its enthalpy, not just internal energy

$$\text{(enthalpy flux)} J_h = (U + PV) \frac{c}{4V} = \frac{4}{3} \sigma T^4 \quad (12)$$

This energy flow process is internally reversible since the photon gas pushed out of  $BB_H$  is in equilibrium with the other particles inside  $BB_H$ , and boundary work is applied to the system to push the photons out in a quasi-equilibrium process. According to the definition of entropy change in classic thermodynamics

$$\Delta S = \left( \frac{\delta Q}{T} \right)_{\text{internally reversible}} \quad (13)$$

where  $\Delta S$  is the entropy change of an equilibrium thermodynamic system and  $\delta Q$  the amount of heat input across the system boundary. The enthalpy flow through the surface area  $A_H$  is equal to the product of the entropy of the photon gas flow and the temperature of the blackbody, and this process is internally reversible. As a result the enthalpy flow across the system boundary  $A_H$  can be treated as heat transfer.

If there is no attenuation of radiation energy between the two blackbodies, the energy absorbed by the low-temperature blackbody ( $BB_L$ ) is the enthalpy of the photon gas leaving the high-temperature blackbody ( $BB_H$ ) according to the conservation of energy principle. On the other hand, the entropy increase in the low-temperature blackbody is not equal to the entropy of the photon gas leaving  $BB_H$ . This is because the photon gas entering  $BB_L$  is not in equilibrium with the other particles in  $BB_L$ . There is entropy gener-

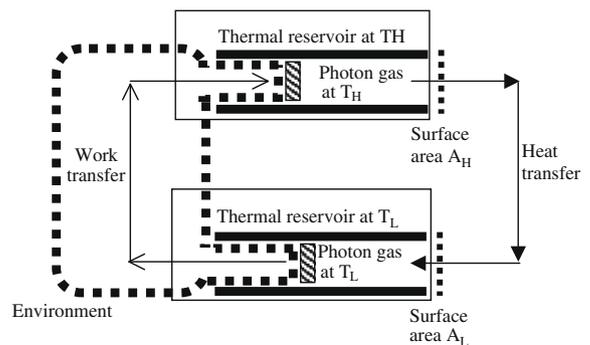


Fig. 2. Energy exchange between two blackbodies at different temperatures ( $T_H > T_L$ ).

ated when the photon gas emitted by  $BB_H$  enters  $BB_L$  and eventually becomes in equilibrium with the other particles in  $BB_L$ . Based on Eq. (3), the final entropy of these photons is

$$S_L = \frac{4}{3} b T_L^3 V_L \quad (14)$$

The increase in entropy in  $BB_L$  can also be computed from the heat input to  $BB_L$

$$S_L = \frac{Q_L}{T_L} = \frac{\frac{4}{3} b T_H^4 V_H}{T_L} \quad (15)$$

Equating Eqs. (14) and (15), we obtain the ratio of the volumes occupied by these photons in the two blackbodies

$$\frac{V_L}{V_H} = \frac{T_H^4}{T_L^4} \quad (16)$$

To push the photon gas of volume  $V_H$  out of  $BB_H$ , boundary work equal to  $P_H V_H$  must be applied to  $BB_H$ . This work is provided by  $BB_L$  for the energy exchange process considered in Fig. 2 in which there is no net energy gain or loss in the environment (the region outside  $BB_H$  and  $BB_L$ ). Inspection of Eqs. (4) and (16) reveals that

$$\frac{P_L V_L}{P_H V_H} = 1 \quad (17)$$

Thus the required boundary work to push the photon gas out of  $BB_H$  is equal to the boundary work produced by the same quantity of photon gas when it enters and expands in  $BB_L$ . This model for photon gas flow and transfer of boundary work is similar to the piston–cylinder device considered in Jeter's analysis [2]. The environment can be considered as full of photons even if it is a vacuum, and work is done on or by the environment when the two pistons in Fig. 2 move.

Similarly the flow of photons at temperature  $T_L$  leaving  $BB_L$  through surface area  $A_L$  can be treated as heat flow since its enthalpy is equal to the product of the photon gas entropy and temperature.  $BB_H$  must do work on  $BB_L$  to push the photon gas of pressure  $P_L$  and volume  $V_L$  out of  $BB_L$ . The net energy transfer rate from  $BB_H$  to  $BB_L$  is therefore the same as that determined from the Stefan–Boltzmann law (for  $A_H = A_L$  and view factor from  $A_H$  to  $A_L = 1$ )

$$(\text{net energy flux from } BB_H \text{ to } BB_L) J_{e, \text{net}} = \sigma(T_H^4 - T_L^4) \quad (18)$$

It should be kept in mind that in the above energy transfer calculation, the heat transfer due to radiation exchange between surfaces  $A_H$  and  $A_L$  is  $4(U_H - U_L)/3$ . The transfer of work required to push photons out of the two thermodynamic systems equals  $-(U_H - U_L)/3$ . It should also be pointed out that, unlike a real gas, the pressure of a photon gas is a function of temperature, not an independent intensive property. The energy transfer rate between the two blackbodies in Fig. 2, which includes transfer of both heat and boundary work, depends only on the temperatures of the two blackbodies.

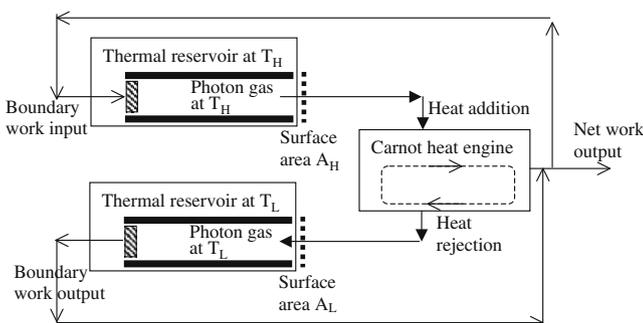


Fig. 3. A Carnot heat engine operating between two blackbodies.

Now consider a Carnot heat engine operating between the two blackbodies as depicted in Fig. 3. The enthalpy flow across surface area  $A_H$  can be modeled as heat transfer as discussed in the preceding paragraphs. This energy flow can be used as the heat input to the Carnot engine. If heat is rejected from the engine in the form of radiation, and the photon gas entering  $BB_L$  is at almost the same temperature as  $T_L$ , the energy flow across  $A_L$  of  $BB_L$  is also heat transfer since the process is internally reversible and the product of entropy flow and temperature is equal to the flow of photon gas enthalpy.

In Fig. 3 the required boundary work for  $BB_H$  can be provided by the heat engine, and the boundary work produced by  $BB_L$  can be added to the engine output. The net work output from the engine is therefore

$$W = \frac{4}{3} U_H \left(1 - \frac{T_L}{T_H}\right) - \frac{1}{3} U_H + \frac{1}{3} U_L = U_H \left(1 - \frac{T_L}{T_H}\right) \quad (19)$$

(where  $U_H$  is the net flow of photon internal energy from  $BB_H$  to the engine) and the conversion efficiency is

$$\begin{aligned} \eta &= (\text{net energy output of the high} \\ &\quad \text{- temperature blackbody}) \\ &\quad / (\text{network output of the engine}) \\ &= 1 - (T_L/T_H) \end{aligned} \quad (20)$$

The above analysis demonstrates that the flow of photons across a system boundary can be treated as heat transfer if the radiation is modeled as cavity radiation and the enthalpy instead of internal energy is used for computing the total energy flux through the cavity opening. Boundary work, which depends on the temperature of radiation for a photon gas, is consumed or produced as the photon gas volume within the system boundary changes, and should be included in the calculation of net energy transfer across the boundaries of the system.

Since heat and work are boundary phenomena and both are observed only at the boundaries of a system [19], the energy flux for radiation transfer within a system (such as in a medium or vacuum) can be simply computed from the flux of photon internal energy, not the enthalpy flux. There is no transfer of boundary work due to the flow of photons within a system. It should also be pointed out that, unlike the transfer of stresses between gas or liquid molecules in a continuum flow, photons do not exert forces on photons since they have no electric charge. However momentum is transferred between photons and a surface when the surface is struck by photons. If the surface is a moving boundary of a system, work can be done on or by the system as demonstrated in the example of a photon-filled, piston–cylinder device in Jeter's paper [2].

An example is given here to illustrate the usefulness of the present model. Consider a furnace cavity that has isothermal walls and a constant and uniform temperature of  $T_H = 1000$  K. The radiation emerging from the opening of the furnace closely resembles that of blackbody radiation at  $T_H$  if the opening is much smaller than the interior surface of the furnace cavity. Thus the enthalpy flux of photon gas leaving the furnace opening, which is equal to the heat flux across the system boundary in the present model, is

$$J_{Q, \text{furnace}} = J_{h, \text{furnace}} = 4\sigma T_H^4/3 = 75,560 \text{ W/m}^2 \quad (21)$$

If a heat engine operating on the Carnot cycle is employed to produce work using radiation from the furnace opening, and the engine rejects heat to the surroundings which are very large and at a uniform temperature of  $T_L = 300$  K, the maximum thermal efficiency of the heat engine is

$$\eta_{\text{max}} = \eta_{\text{Carnot}} = 1 - (T_L/T_H) = 0.7 \quad (22)$$

If the view of the furnace opening is totally blocked by the engine, and the engine surface is black and at  $T_H - dT$  during heat addition, the net rate of heat transfer from the furnace to the engine is

$$\begin{aligned} dQ_H/dt &= J_{Q,H} A_H = (J_{Q,\text{furnace}} - J_{Q,\text{engine}}) A_H \\ &= 4A_H \sigma [T_H^4 - (T_H - dT)^4] / 3 \end{aligned} \quad (23)$$

where  $A_H$  is the surface area of the furnace opening. For  $dT = 0.1$  K (strictly speaking, the temperature difference during heat transfer in the Carnot cycle should be infinitesimal) and  $A_H = 0.01$  m<sup>2</sup>, the net rate of heat input to the engine is

$$dQ_H/dt = 0.3022 \text{ W} \quad (24)$$

For the heat input rate given in Eq. (24), the maximum power output of the heat engine is

$$dW_{\text{engine, max}}/dt = dQ_H/dt \quad \eta_{\text{max}} = 0.2115 \text{ W} \quad (25)$$

The net rate of boundary work transferred to the photons in the furnace cavity, which is modeled as the high-temperature piston-cylinder device in Fig. 3, can be calculated from

$$\begin{aligned} -dW_H/dt &= -A_H \sigma [T_H^4 - (T_H - dT)^4] / 3 \\ &= -0.0755 \text{ W} \quad (\text{In the present analysis, work done by} \\ &\quad \text{a system is defined as positive.}) \end{aligned} \quad (26)$$

The net rate of energy flow out of the furnace is therefore the Stefan-Boltzmann law

$$dQ_H/dt - dW_H/dt = A_H \sigma [T_H^4 - (T_H - dT)^4] = 0.2267 \text{ W} \quad (27)$$

The heat rejection rate of the Carnot heat engine is

$$dQ_L/dt = dQ_H/dt (T_L/T_H) = 0.0907 \text{ W} \quad (28)$$

If heat rejection from the engine to the surroundings is by radiation alone through an aperture of surface area  $A_L$ , and  $A_L$  is much smaller than the surroundings, the aperture can be approximated as a blackbody at  $T_L$ . In this case the heat rejection rate can be expressed as

$$dQ_L/dt = J_{Q,L} A_L = 4A_L \sigma [(T_L + dT)^4 - T_L^4] / 3 \quad (29)$$

In the above equation  $dT$  is the temperature difference between the engine surface and the surroundings during heat rejection. The required aperture surface area for the low-temperature blackbody can be determined from Eqs. (28) and (29). The result is

$$A_L = 0.111 \text{ m}^2 \quad (30)$$

Since the surroundings are modeled as the low-temperature, piston-cylinder device in Fig. 3, the net boundary work it can produce (per unit time) during heat rejection is

$$dW_L/dt = A_L \sigma [(T_L + dT)^4 - T_L^4] / 3 = 0.0227 \text{ W} \quad (31)$$

The maximum net power output for the system depicted in Fig. 3, which consists of a Carnot heat engine and two photon-filled, piston-cylinder devices, is

$$dW_{\text{net, max}}/dt = dW_{\text{engine, max}}/dt - dW_H/dt + dW_L/dt = 0.1587 \text{ W} \quad (32)$$

The above calculated power output is equal to the one determined from the product of the internal energy flow rate (the Stefan-Boltzmann law) and the Carnot cycle efficiency

$$dW_{\text{net, max}}/dt = A_H \sigma [T_H^4 - (T_H - dT)^4] \eta_{\text{Carnot}} = 0.1587 \text{ W} \quad (33)$$

## Acknowledgment

This work was partially supported by the New and Renewable Energy Center of KEMCO under Project No. 2006-N-S008-01-0-000.

## References

- [1] Landsberg PT, Tonge G. Thermodynamic energy conversion efficiencies. *J Appl Phys* 1980;51(7):R1–R20.
- [2] Jeter SM. Maximum conversion efficiency for the utilization of direct solar radiation. *Sol Energy* 1981;26:231–6.
- [3] Castans M. Comments on 'Maximum conversion efficiency for the utilization of direct solar radiation'. *Sol Energy* 1983;30(3):293.
- [4] De Vos A, Pauwels H. Letters to the editor: comment on a controversy between M. Castans and S. Jeter. *Sol Energy* 1984;33(1):91–3.
- [5] Gribik JA, Osterle JF. The second law efficiency of solar energy conversion. *ASME J Sol Energy Eng* 1984;106:16–21.
- [6] Boehm RF. Maximum performance of solar heat engine. *Appl Energy* 1986;23(4):281–96.
- [7] Gyftopoulos EP, Beretta GP. *Thermodynamics: foundations and applications*. New York: Macmillan; 1991. p. 428–67.
- [8] Petela R. Exergy of undiluted thermal radiation. *Sol Energy* 2003;74:469–88.
- [9] Badescu V. On the theoretical maximum efficiency of solar-radiation utilization. *Energy* 1989;14(9):571–3.
- [10] Badescu V. Maximum conversion efficiency for the utilization of multiply scattered solar radiation. *J Phys D: Appl Phys* 1991;24:1982–5.
- [11] Badescu V. Letter to the editor. *Sol Energy* 2004;76:509–11.
- [12] Humphrey TE, Linke H. Quantum, cyclic, and particle-exchange heat engines. *Physica E* 2005;29:390–8.
- [13] Laptev VI. Solar energy conversion with and without work production: a supplement to the actual thermodynamic description. In: IEEE 4th world conference on photovoltaic energy conversion, 2006. p. 91–4.
- [14] Incropera FP, Dewitt DP, Bergman TL, Lavine AS. *Fundamentals of heat and mass transfer*. 6th ed. New York: John Wiley & Sons; 2007. p. 1–55.
- [15] Saad MA. *Thermodynamics – principles and practice*. New Jersey: Prentice-Hall; 1997. p. 703–850.
- [16] Carter AH. *Classical and statistical thermodynamics*. New Jersey: Prentice-Hall; 2001. p. 333–40.
- [17] Cengel YA, Boles MA. *Thermodynamics: an engineering approach*. 6th ed. New York: McGraw-Hill; 2008. p. 51–92.
- [18] Bejan A. Unification of three different theories concerning the ideal conversion of closed radiation. *ASME J Sol Energy Eng* 1987;109:46–51.
- [19] Van Wylen GJ, Sonntag RE, Borgnakke C. *Fundamentals of classical thermodynamics*. 4th ed. New York: John Wiley & Sons; 1994. p. 85.