Design for Reliability

MEEM 5401

Reliability Analysis for Vehicle Side Impact Protection to Verify the Response of Rib Deflection and The Pubic Symphysis Force

<u>Project 1</u>

Submitted by

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Abstract

Reliability analysis using three approaches for vehicle side impact protection to verify the response of rib deflection and the pubic symphysis force are presented. The first approach is the direct MCS method which is based in the crude sampling and is considered as the benchmark method for comparing with other two approaches. The second approach is the FORM which is based on linearizing the performance function and the third approach is the UDR method which is based on converting a multidimension function into multiple one-dimension functions. The results of FORM and UDR method are compared with the direct MCS method and the errors are computed. The advantages and limitations of these approaches are discussed.

1. Introduction

As we know we are here in 21st century and with the advancement in computer technology the vehicle safety issue has been dealt with computational methods which allow us to optimize the design with less cost and time. The vehicle design department also requires a multidisciplinary view of optimization along with computational methods. But simulation-based methods produce deterministic models that are optimized leaving no space for uncertainties or imperfections in manufacturing or tolerances in design. This results in the formation of designs that are not reliable. This leads to a requirement of methods such as reliability-based design methods.

The problem statement of our project includes an evaluation of the reliability of responses for a vehicle using first-order reliability method and univariate dimension reduction method. We have followed European enhances vehicle safety committee side impact test procedure for our project.

The objectives of our project involve determining the reliability of the vehicle side impact test using FORM and UDR. Our problem has two test functions which are given in the question and two limit state functions. Design variables involve in test function includes parameters like roof rail, cross member various pillars and so on. Another objective would be to judge the performance of FORM and the UDR method under the influence of normally distributed data. Our last objective is to compare results obtained from these two methods with Monte Carlo simulation results.

Responses given in the project:

$$Deflection_{rib} = 28.98 + 3.818d_3 - 4.2d_1d_2 + 0.0207d_5x_1 + 6.63d_6d_9 - 7.7d_7d_8 + 0.32d_9x_1$$
$$Force_{pubic\ symphysis} = 4.72 - 0.5d_4 - 0.19d_2d_3 - 0.0122d_4x_1 + 0.009325d_6x_1 + 0.000191x_2^2$$

The d_1 to d_9 and x_1, x_2 are the design variables given for the problem. These are the various parts of the vehicle which are briefly mentioned in Table 1.1. The mean and standard deviation were given for this problem for the normally distributed data. In Table 1.2., the altered parameters are given referred from Lee [1] for more accurate results.

Design variables	Standard deviation	Mean	Distribution
d_1(B-pillar inner)	0.03	0.8262	Normal
d_2(B-pillar reinforce)	0.03	1.3288	Normal
d_3(Floor side inner)	0.03	0.5	Normal
d_4(Cross member)	0.03	1.5	Normal
d_5(Door beam)	0.03	0.5	Normal
d_6(Door belt line)	0.03	1.1423	Normal
d_7(Roof rail)	0.03	0.9093	Normal
d_8(Mat. floor side inner)	0.006	0.3035	Normal
d_9(Mat. floor side)	0.006	0.2294	Normal
x_1(Barrier height)	10	0	Normal
x_2(Barrier hitting)	10	0	Normal

Table 1.1. Given Parameters

Design variables	Standard deviation	Mean	Distribution
d_1(B-pillar inner)	**0.10**	0.8262	Normal
d_2(B-pillar reinforce)	**0.10**	1.3288	Normal
d_3(Floor side inner)	**0.10**	0.5	Normal
d_4(Cross member)	**0.10**	1.5	Normal
d_5(Door beam)	**0.10**	0.5	Normal
d_6(Door belt line)	**0.10**	1.1423	Normal
d_7(Roof rail)	**0.10**	0.9093	Normal
d_8(Mat. floor side inner)	0.006	0.3035	Normal
d_9(Mat. floor side)	0.006	**0.60**	Normal
x_1(Barrier height)	10	0	Normal
x_2(Barrier hitting)	10	0	Normal

Table 1.2. Modified Parameters

Limit State Functions:

$$G_1 = 32 - Deflection_{rib} \quad ; G_2 = 4 - Force_{pubic}$$

In the given limit state functions, 32 is limiting value for Rib Deflection and 4 is limiting value for Pubic Force. The limit state function differentiates the safe region and failure region in the results. It returns to the negative value if system fails and a positive value if the system is stable.

2. Reliability Analysis

2.1 Monte-Carlo Simulation Method

The Monte-Carlo simulation follows the flow chart shown below:

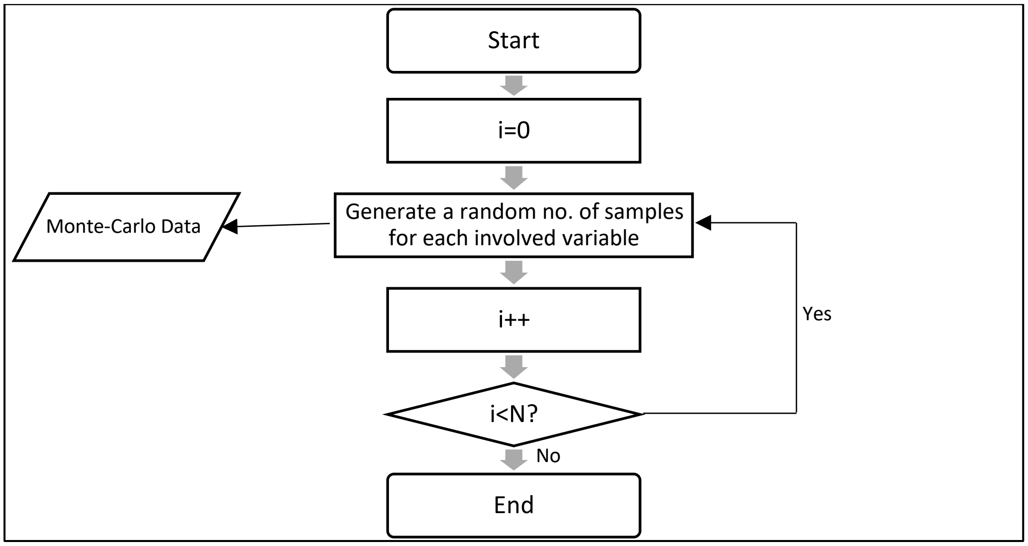

Fig. 2.1.1. Flow chart of Monte-Carlo simulation [3]

The Monte-Carlo simulation is most crude sampling method, yet it is widely used for sampling due to its accuracy. In MCS, large no. of values of system performances were generated using random samples based on the normal distribution models. Then, the limit state function is used to map the safe and failure region. The reliability is the ratio of no. of samples in the safe region to the total no. of samples.

In this project,

Total No. of Samples (N) = 10^5

The results obtained from the MCS are shown in fig. 2.1.2 and 2.1.3.

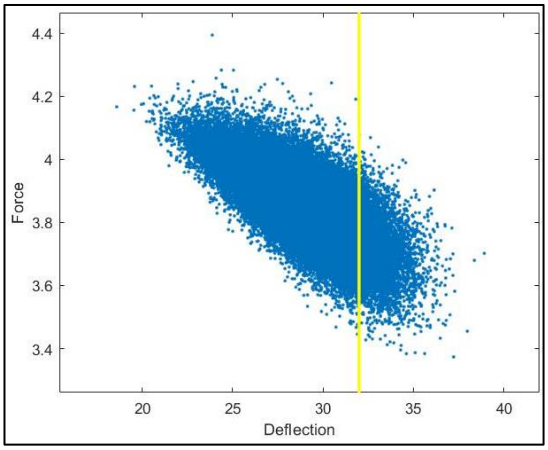

Fig. 2.1.2. Limit State function for Deflection

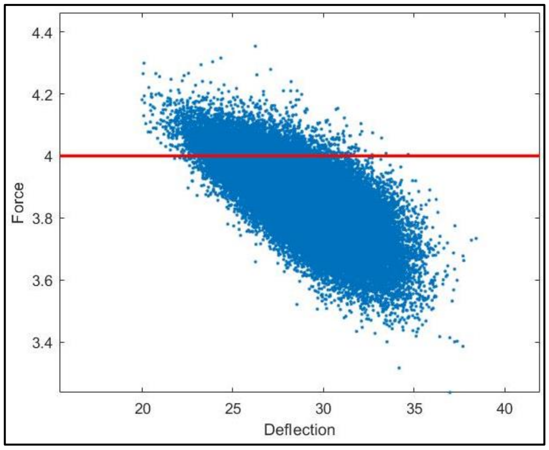

Fig. 2.1.3. Limit State function for Force

The reliabilty obtained from given parameters were close to absolute value for the rib deflection, so when we tried with modified values, the reliability for the deflection obtained as 93.24% and 92.24% for force.

Responses	Original Question	Modified Values
Deflection	100%	93.24%
Force	95.03%	92.24%

Table 2.1. Results of Reliability by MCS

Although, the Monte-Carlo simulation method is simple, but it is bit time-consuming process. The generation of random numbers for each system performance is very lengthy process, especially for the problem used in this project which does have 11 system variables. This makes the system as 11-dimensional problem.

2.2 <u>First Order Reliability Method</u>

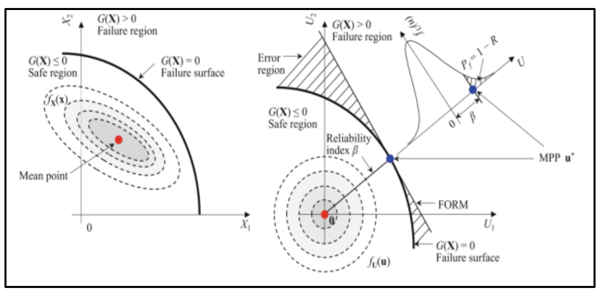

Fig. 2.2.1. Transformation from X-space to U-space [4]

The basic idea of FORM is to linearize the performance function G(x) at the most probable failure point on the limit state surface. In this method the input random variables in the original X-space is transformed to the U-space. There are generally two approaches in FORM method, the Reliability index approach (RIA) and Performance measure approach (PMA). For this project RIA approach is used as the project demands reliability analysis as the PMA approach is most suitable for design optimization problems. In RIA method reliability is calculated as, $R = \Phi(\beta)$ where, beta is the minimum distance between the Most probable point (MPP) point and the origin. This method of reliability analysis is suitable for linear failure surface transformation in U-space. It gives exact reliability values. But, it overestimates the reliability for non-linear transformation. In most of the engineering applications the failure surface transformation is non-linear and thus this is the major limitation of this method.

In this project the data generated from the input random variables is normally distributed and thus the transformation from X-space to U-space can be computed by using, $X = \mu + \sigma U$ After transformation of input random variables the critical task in FORM method is to search for MPP points. The MPP search is an iterative optimization scheme process based on gradient information of the performance function. In this project the Hasofer-Lind and Rackwitz-Fiessler (HL-RF) algorithm is used for MPP search as this method is simple and has good efficiency.

In HL-RF algorithm, the initial MPP estimate is at the mean value. Then the performance function is computed and its partial derivatives with respect to the input random variables in the U-space. Then, it updates the search point at the current iteration as,

$$\mathbf{u}^{(k+1)} = \left(\mathbf{u}^{(k)} \cdot \mathbf{n}^{(k)} - \frac{G(\mathbf{u}^{(k)})}{\|\nabla_{\mathbf{U}} G(\mathbf{u}^{(k)})\|}\right)\mathbf{n}^{(k)}$$
$$= \left[\mathbf{u}^{(k)} \cdot \nabla_{\mathbf{U}} G\left(\mathbf{u}^{(k)}\right) - G\left(\mathbf{u}^{(k)}\right)\right] \frac{\nabla_{\mathbf{U}} G(\mathbf{u}^{(k)})}{\|\nabla_{\mathbf{U}} G(\mathbf{u}^{(k)})\|^2}$$

And continues the process until convergence. In this project the difference between the current MPP point and the MPP point in the next iteration is specified as 1*10^-5 to see the convergence and the total number of iterations to be less than twenty. The number of iterations required in this project for rib deflection is three and for pubic symphysis force is five. To compute reliability for the responses, the minimum of beta is selected and the values obtained for reliability are;

Responses	Original Question	Modified Values	No. of Evaluation
Deflection	100%	93.25%	36
Force	97.81%	96.21%	60

Table 2.2. Results of Reliability by FORM

In FORM the number of evaluation for each response were calculated as:

No. of evaluation=(No. of design variables+ No. limit state functions)*(No. of iterations)

Thus, for rib deflection the No. of evaluations=(11+1)*3= 36

For pubic symphysis force the No. of evaluations=(11+1)*5= 60

2.3 <u>Univariate Dimension Reduction Method</u>

Dimension reduction method is known as the Higher Dimensional Model Representation (HDMR) method. HDMR basically is a univariate dimension reduction method which simplifies one dimensional response surface to multiple one-dimensional component function. Dimension reduction method is one of the stochastic response surface method which are capable of reducing the computational burden required by the sampling methods while still maintaining a good accuracy.

Steps involved in UDR reliability analysis:

i. Generate Normally distributed random data for the variables.
ii. Decomposing Multidimensional Performance function into multiple one-dimensional univariate functions.
iii. Using Cubic Spline interpolation to approximate the one-dimensional univariate functions over Empirical sample point (m=**7**). i.e. the number of times the function was evaluated is equal to 67. (Total Number of Dimension(11) * 6 + 1)
iv. Using Monte Carlo simulation along with the Stochastic response surface to obtain the reliability of the performance function.

Advantages of UDR method here is that MCS is employed on the explicit interpolation instead of the original performance function due to which it becomes economical and requires less time. One more added benefit is the approximation of the response function over the input domain allows for the derivation of any probabilistic characteristics suggest artistical movements reliability and probability density function.

Responses	Reliability		% Error
	Direct MCS	UDR	
Deflection	93.24%	93.14%	0.107%
Force	92.24%	91.67%	0.617%

Table 2.3. Results of Reliability by UDR

3. <u>Results</u>

Table below shows the reliability values of the three approaches used for reliability analysis for this project.

Performance	Reliability		
	Direct MCS	FORM method	UDR method
Rib Deflection	93.24%	93.25%	93.14%
Force	92.24%	96.21%	91.67%

Table 3.1. Overall Comparison of Reliability

The PDF plot shown below shows the comparison between the MCS and UDR method.

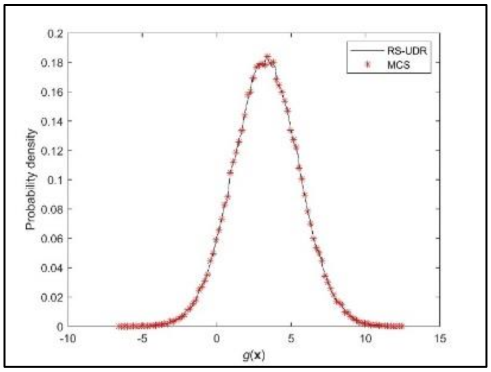

Fig. 3.1 PDF for Rib Deflection

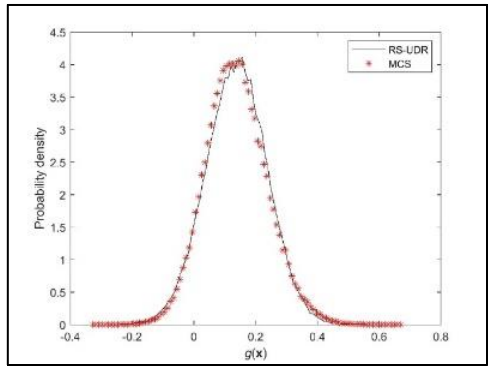

Fig. 3.2 PDF for pubic symphysis force

4. Conclusion

We compared reliability from FORM and UDR method with the Direct MCS method. We found that FORM method gave exact reliability values for the rib deflection response but it overestimated the reliability for pubic symphysis force. This overestimation of reliability is a result of nonlinear transformation of limit state function as the limit state function was quadratic. Comparing MCS with UDR we found that there was not much difference in the reliability values. But the time required for direct MCS reliability analysis was more than the UDR method. Thus, for our project UDR method is most suitable for reliability analysis.

References

[1] Lee, Ikjin & Choi, Kyung & Du, Liu & Gorsich, David. (2008). Dimension reduction method for reliability-based robust design optimization. Computers & Structures. 86. 1550-1562. 10.1016/j.compstruc.2007.05.020.

[2] C. Hu, B.D. Youn, P. Wang, *Engineering Design Under Uncertainty and Health Prognostics* (Springer, Cham, 2018). ISBN 978-3-319-92572-1

[3] “An Introduction to Reliability Analysis” from https://risk-reliability.uniandes.edu.co/wiki/doku.php

[4] C. Hu, B.D. Youn, P. Wang, *Engineering Design Under Uncertainty and Health Prognostics* (Springer, Cham, 2018). ISBN 978-3-319-92572-1 (Page-109)