processes [27] unless otherwise stated. [Appendix D contains a greater explanation of the Scilab Code.]

Table 3.21.1-1: Nomenclature

a	Modified ideality factor (V)	**Greek Symbols**	
A	Area (m^2)	β	Collector tilt (degree)
C_b	Bond Conductance (W/m)	δ	Declination: sun's angular position at noon with respect to the plane of the equator
C_p	Specific heat capacity of air (kJ/kg K)	Δ	Difference in temperature, pressure
D	Diameter (m), pipe diameter (m)	ε	Emissivity
E_g	Material bandgap (eV)	η	Efficiency
Ex	Exergy	μ	Viscosity (kg/s m)
f_r	Friction factor	$\mu_{I,sc}$	Short circuit current temperature coefficient
F	Fin efficiency factor	ρ	Density (kg/m^3)
F'	Collector efficiency factor	σ	Stefan-Boltzmann's constant (W/m^2 K^4)
F''	Collector flow factor	ϕ	Latitude of the location being studied
F_R	Collector heat removal factor	***Subscripts***	
h	Heat transfer coefficient (W/m^2 K)	1	Length
I	Current (A)	2	Width
IV	Current voltage	amb	Ambient
k	Thermal conductivity (W/m K) , Boltzmann's constant (m^2 kg /s^2 K)	b	Back
L	Dimensions of the solar module, length of system, thickness, duct length (m)	$cell$	Cell
$\dot{m}$	Air mass flow rate (Kg/s)	f	Fluid
N	Number of glass covers	g	Glass
Nu	Nusslet number	h	Hydraulic
p	Flow pressure (Pa)	in	Inlet
P	Perimeter (m)	i	Inner
PV	Photovoltaic	L	Loss, Light
PV/T	Photovoltaic solar thermal hybrid s	m	Mean
Q_u	Useful gain (W)	mp	Maximum power point
R	Resistance (Ω)	o	Reverse saturation
Re	Reynolds number	oc	Open circuit
S	Solar radiation intensity (W/m^2)	p	Panel

T	Temperature (K)	*pv*	Photovoltaic
U_b	Overall back loss coefficient (W/m^2 K)	*pv/t*	Photovoltaic solar thermal system
U_e	Overall edge loss coefficient (W/m^2 K)	*r*	Radiation
U_L	Overall loss coefficient (W/m^2 K)	*ref*	Reference
U_t	Overall top loss coefficient (W/m^2 K)	*s*	Series
V	Voltage (V) , Velocity (m/s)	*sc*	Short circuit
W	Distance between diameter of pipe (m)	*sh*	Shunt
y	Empirically determined coefficient establishing the upper limit for module temperature at low wind speeds and high solar irradiance	*th*	Thermal
z	Empirically determined coefficient establishing the rate at which module temperature drops as wind speed increases	*w*	Wind

3.12.2 PV Model

The starting equation for the model of the solar cell describes the solar cell as a diode and can be seen in Equation 3.11.2-1.

$$I = I_L - I_D - \frac{V + IR_s}{R_{sh}} = I_L - I_o\left[e^{\frac{V+IR_s}{a}} - 1\right] - \frac{V + IR_s}{R_{sh}} \qquad \text{Eq 3.12.2-1}$$

Where I is the current, I_L is the leakage current, I_o is the reverse saturation current, V is the voltage, R_s is the series resistance and a is the modified ideality factor. A circuit depiction of Equation 3.11.2-1 can be found in Figure 3.12.2-1.

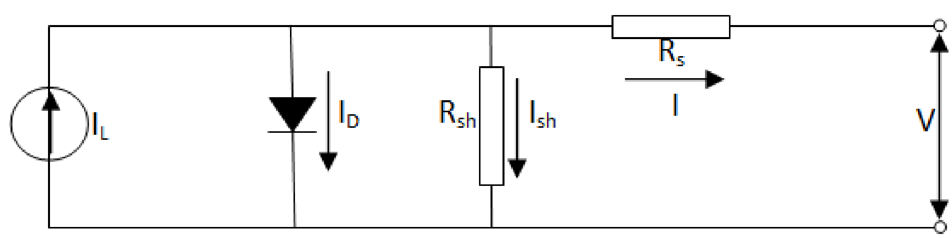

Figure 3.12.2-1: Five Parameter Photovoltaic Model Equivalent Electric Circuit

To solve for the five parameters, the initial conditions were applied to Equation 3.12.2-1. At the short circuit current conditions, the current, I, is equal to the reference short circuit current ($I_{sc,\,ref}$)

and the voltage is equal to zero. Furthermore, the slope of the current with respect to the voltage is equal to the negative inverse of the shunt resistance (R_{sh}). In the open circuit conditions, the current equals zero and the voltage equals the reference open circuit voltage ($V_{oc,ref}$). At the maximum power condition, the current equals the reference maximum power current ($I_{mp,ref}$) and the voltage equals the reference maximum power voltage ($V_{mp,ref}$). Furthermore the change in the maximum power is zero.

When these conditions are applied to the diode Equation, eq. 3.12.2-1, the following five equations are produced (3.12.2-2 to 3.12.2-6).

$$I_{sc,ref} = I_{L,ref} - I_{o,ref}\left[e^{\frac{I_{sc,ref}R_{s,ref}}{a_{ref}}} - 1\right] - \frac{I_{sc,ref}R_{s,ref}}{R_{sh,ref}}$$ Eq 3.12.2-2

$$\frac{1}{R_{sh,ref}} = \frac{\frac{I_{o,ref}}{a_{ref}}\left[e^{\frac{I_{sc,ref}R_{s,ref}}{a_{ref}}} - 1\right] - \frac{1}{R_{sh,ref}}}{1 + \frac{I_{o,ref}R_{s.ref}}{a_{ref}}\left[e^{\frac{I_{sc,ref}R_{s,ref}}{a_{ref}}} - 1\right] - \frac{R_{s,ref}}{R_{sh,ref}}}$$ Eq 3.12.2-3

$$0 = I_{L,ref} - I_{o,ref}\left[e^{\frac{V_{sc,ref}}{a_{ref}}} - 1\right] - \frac{V_{sc,ref}}{R_{sh,ref}}$$ Eq 3.12.2-4

$$I_{mp,ref} = I_{L,ref} - I_{o,ref}\left[e^{\frac{V_{mp,ref}+I_{mp,ref}R_{s,ref}}{a_{ref}}} - 1\right] - \frac{V_{mp,ref}+I_{mp,ref}R_{s,ref}}{R_{sh,ref}}$$ Eq 3.12.2-5

$$\frac{I_{mp,ref}}{V_{mp,ref}} = \frac{\frac{I_{o,ref}}{a_{ref}}\left[e^{\frac{V_{mp,ref}+I_{mp,ref}R_{s,ref}}{a_{ref}}} - 1\right] - \frac{1}{R_{sh,ref}}}{1 + \frac{I_{o,ref}R_{s.ref}}{a_{ref}}\left[e^{\frac{V_{mp,ref}+I_{mp,ref}R_{s,ref}}{a_{ref}}} - 1\right] - \frac{R_{s,ref}}{R_{sh,ref}}}$$ Eq 3.12.2-6

Solving Equations 3.12.2-2 to 3.12.2-6 produces the reference values for the I_o, I_L, a, R_s and R_{sh}. These variables are then used to calculate the operating condition values. The equations used to solve for the operating values are the following eq. 3.12.2-7 to 3.12.2-11.

$$\frac{a}{a_{ref}} = \frac{T_{cell}}{T_{cell,ref}} \qquad \text{Eq 3.12.2-7}$$

where T_{cell} is the PV cell's temperature in Kelvins.

$$\frac{R_{sh}}{R_{sh,ref}} = \frac{S_{ref}}{S} \qquad \text{Eq 3.12.2-8}$$

where S is the irradiance in W/m^2.

$$I_L = \frac{S}{S_{ref}}\left[I_{L.ref} + \mu_{I,sc}\left(T_{cell} - T_{cell,ref}\right)\right] \qquad \text{Eq 3.12.2-9}$$

Where μ_{Isc} is the current temperature coefficient in A/°C.

$$E_g = E_{g,ref}\left[1 - C\left(T_{cell} - T_{cell,ref}\right)\right] \qquad \text{Eq 3.12.2-10}$$

Where E_g is the band gap of the solar cell in electron volts (eV). In this case, it is the band gap of silicon.

$$I_o = I_{o,ref}\left[\left(\frac{T_{cell}}{T_{cell,ref}}\right)^3 e^{\left(\left[\frac{E_{g,ref}}{kT_{cell,ref}}\right]-\left[\frac{E_g}{kT_{cell}}\right]\right)}\right] \qquad \text{Eq 3.12.2-11}$$

R_s is assumed to be independent of both temperature and irradiance. These variable results allowed for the calculation of V_{oc}, I_{sc}, V_{mp} and I_{mp}. The equations 3.12.2-12 and 3.12.2-13 were used to solve V_{oc} and I_{sc} and the V_{mp} and I_{mp}. For solving for the I_{sc} and V_{oc} just replace the V_{mp} with the V_{oc} and the I_{mp} with the I_{sc}.

$$I_{mp} = I_L - I_o\left[e^{\frac{V_{mp}+I_{mp}R_s}{a}} - 1\right] - \frac{V_{mp} + I_{mp}R_s}{R_{sh}} \qquad \text{Eq 3.12.2-12}$$

$$\frac{I_{mp}}{V_{mp}} = \frac{\frac{I_o}{a}\left[e^{\frac{V_{mp}+I_{mp}R_s}{a}}-1\right]-\frac{1}{R_{sh}}}{1+\frac{I_o R_s}{a}\left[e^{\frac{V_{mp}+I_{mp}R_s}{a}}-1\right]-\frac{R_s}{R_{sh}}} \qquad \text{Eq 3.12.2-13}$$

The V_{mp} was multiplied by the I_{mp} to calculate the power produced by the cell under operating conditions.

The Petela derived total solar exergy entering the system was used and is given Equation 3.12.2-14 [31].

$$\dot{E}x_{in} = \left(1-\frac{3}{4}\frac{T_{amb}}{T_{sun}}+\frac{1}{3}\left(\frac{T_{amb}}{T_{sun}}\right)^4\right)SA_p \qquad \text{Eq 3.12.2-14}$$

Where the T_{amb} and T_{sun} are the ambient and sun temperature in Kelvin.

3.12.3 Solar Panel Temperature

To solve for the temperature of the cell at operating temperatures, the empirically derived equation from the Sandia National Laboratory was used. The error in the equation is ±5 [33]. The module used to describe the panel in this simulation was a glass/cell/ polymer sheet on an open mount. To determine the temperature of the back of the model Equation 3.12.3-1 was used [33].

$$T_m = Se^{y+zV_w} + T_{amb} \qquad \text{Eq 3.12.3-1}$$

Where y is dimensionless and z is s/m, are the empirically determined coefficients with the values of -3.56 and -0.075 for the glass/cell/polymer sheet open rack module type and V_w is the wind velocity is m/s. To determine the temperature of the cell, Equation 3.12.3-2 was implemented [33].

$$T_{cell} = T_m + \frac{S}{S_{ref}}\Delta T \qquad \text{Eq 3.12.3-2}$$

Where ΔT is the temperature difference between the panel's back surface (T_m) and the cell's temperature at an irradiance value of (S_{ref}) 1000 W/m^2. In the case of the module being considered, the temperature difference value is 3°C.

3.12.4 Flat Plate Collector Model

The thermal mode of a solar flat plate uses the equations given by Duffie and Beckman [27]. To determine the efficiency of the solar collector, the overall heat loss from the system is needed. The overall heat loss of the system was calculated using equations 3.12.4-1 to 3.12.4-7. Equation 3.12.4-1 was employed to calculate the top heat losses (U_t).

$$U_t = \left(\frac{N}{\frac{C}{T_{pm}}\left[\frac{T_{pm}-T_{amb}}{N+f}\right]^e} + \frac{1}{h_w} \right)^{-1} + \frac{\sigma\left(T_{pm}^2 + T_{amb}^2\right)\left(T_{pm}+T_{amb}\right)}{\frac{1}{\varepsilon_p + 0.00591Nh_w} + \frac{2N+f-1+0.133\varepsilon_p}{\varepsilon_g} - N} \quad \text{Eq 3.12.4-1}$$

Where N is the number of glass covers, T_{pm} and T_{amb} are the temperature of the plate and ambient temperature (K), β is the collector tilt (degrees), ε_g and ε_p are the emissivity of the glass and plate and h_w is the wind heat transfer coefficient (W/m^2 C) which can be found using Equation 3.12.4-2 [40].

$$h_w = 2.8 + 3V_w \quad \text{Eq 3.12.4-2}$$

The coefficients f, C, and e in eq. 3.12.4-1 are calculated using equations 3.12.4-3 to 3.12.4-5.

$$f = \left(1 + 0.089h_w - 0.1166h_w\varepsilon_p\right)\left(1 + 0.07866N\right) \quad \text{Eq 3.12.4-3}$$

$$C = 520\left(1 - 0.000051\beta^2\right) \quad \text{Eq 3.12.4-4}$$

$$e = 0.43\left(1 - \frac{100}{T_{pm}}\right) \quad \text{Eq 3.12.4-5}$$

The side, U_b, and bottom, U_e, losses were calculated using equations 3.11.4-6 and 3.11.4-7.

$$U_b = \frac{k}{L} \quad \text{Eq 3.12.4-6}$$

$$U_e = \frac{\left(\frac{k}{L}A\right)_{edge}}{A_p} \qquad \text{Eq 3.12.4-7}$$

Where k is the thermal conductivity (W/m K), L is the thickness (m) and A is the area (m^2).

The total loss of the system is the sum of U_L, U_b and U_e as in Equation 3.12.4-8.

$$U_L = U_b + U_e + U_t \qquad \text{Eq 3.12.4-8}$$

Using U_L the fin collector efficiency factor F' was calculated using Equation 3.12.4-9 [41-43].

$$F' = \frac{\frac{1}{U_L}}{W\left[\frac{1}{U_L[D+(W-D)F]} + \frac{1}{C_b} + \frac{1}{\pi D_i h_f}\right]} \qquad \text{Eq 3.12.4-9}$$

Where W is the pipes center to center distance (m), D and D_i is the outer and inner diameter of the pipe (m), C_b is the bond conductance which is assumed to be very large ($\frac{1}{C_b} = 0$) (W/m K), h_f is the heat transfer coefficient between the fluid and the pipe wall (W/K) which can be calculated using Equation 3.12.4-10.

$$h_f = Nu\frac{k}{D_h} \qquad \text{Eq 3.12.4-10}$$

Where Nu is the Nusselt number found by using Equation 3.12.4-11. Equation 3.12.4-11 was derived for a fully developed turbulent airflow with one side heated and the other side insulated [27].

$$Nu = 0.0158\,\mathrm{Re}^{0.8} \qquad \text{Eq 3.12.4-11}$$

Where Re is the Reynolds number which can be calculated using Equation 3.12.5-5 found in Section 3.12.5.

F is the fin efficiency factor which can be calculated from Equation 3.12.4-12 [41-43].

$$F = \frac{\tanh[m(W - D)/2]}{m(W - D)/2} \qquad \text{Eq 3.12.4-12}$$

Where m can be calculated using Equation 3.11.4-.13.

$$m = \sqrt{\frac{U_L}{\delta k}} \qquad \text{Eq 3.12.4-13}$$

Where δ is the thickness of the plate (m) and k is the thermal conductivity of the plate (W/m K). Using the fin collector efficiency factor F' found by Equation 3.12.4-9, the heat removal factor was determined from Equation 3.12.4-14 [41-43].

$$F_R = \frac{\dot{m}C_p}{A_P U_L}\left(1 - e^{-\frac{A_p U_L F'}{\dot{m}C_p}}\right) \qquad \text{Eq 3.12.4-14}$$

The actual useful energy gain Q_u was then calculated using the Equation 3.12.4-15.

$$Q_u = A_p F_R\left[S - U_L\left(T_{in} - T_{amb}\right)\right] \qquad \text{Eq 3.12.4-15}$$

Using the Q_u the mean temperature of the plate and the fluids outflow were calculated with equations 3.12.4-16 and 3.12.4-17 [41-43].

$$T_{pm} = T_{in} + \frac{Q_u / A_p}{U_L F_R}\left(1 - F_R\right) \qquad \text{Eq 3.12.4-16}$$

$$T_{out} = T_{in} + \frac{Q_u}{\dot{m}C_p} \qquad \text{Eq 3.12.4-17}$$

3.12.5 Thermal Exergy

The change in exergy for the thermal system is derived from the difference in the exergy of the flow at the inlet and outlet [22-24]. This is given by the following equation.

$$\Delta\dot{E}x_{th} = \dot{m}C_p\left(T_{out} - T_{in} - T_{amb}\ln\left(\frac{T_{out}}{T_{in}}\right)\right) - \frac{\dot{m}T_{amb}\Delta p}{\rho T_{in}} \qquad \text{Eq 3.12.5-1}$$

Where $\dot{m}$ is the mass flow rate (kg/s), C_p is the specific heat capacity (s/Kg *K), T_{out}, T_{in} and T_{amb} are the outlet, inlet and ambient temperature (K), ρ is the density (kg/m^3) and Δp is the frictional pressure drop (Pa)

The frictional pressure drop of the fluid Δp was calculated using Equation 3.12.5-2

$$\Delta p = f_r \rho L \frac{V^2}{2D_h} \qquad \text{Eq 3.12.5-2}$$

Where L is the length of the duct (m), V is the velocity (m/s), D_h is the hydraulic diameter as seen in Equation 3.12.5-3.

$$D_h = \frac{4A_f}{P_f} \qquad \text{Eq 3.12.5-3}$$

Where A is the area (m^2) and P is the perimeter (m) and f is the friction which can be calculated using Equation 3.12.5-4.

$$f_r(\mathrm{Re}) = \begin{cases} \dfrac{64}{\mathrm{Re}} & ,\mathrm{Re} \le 2200 \\ 0.316\,\mathrm{Re}^{-0.25} & ,\, otherwise \end{cases} \qquad \text{Eq 3.12.5-4}$$

Where Re is the Reynolds number that was calculated from Equation 3.12.5-5

$$\mathrm{Re} = \frac{\dot{m} D_h}{A_f \mu} \qquad \text{Eq 3.12.5-5}$$

Where μ is the Viscosity (kg/s m).

The total change in exergy given to the system by the sun for the thermal exergy analysis can be seen in Equation 3.12.5-6. This equation was derived from the Jeter analysis and is based on the Carnot cycle [31, 32].

$$\dot{E}x_{in} = \left(1 - \frac{T_{amb}}{T_{sun}}\right) S A_p \qquad \text{Eq 3.12.5-6}$$

3.12.6 PVT Model Thermal Aspect

The thermal aspect of the PVT model uses the equations found in Section 3.12.4 with the following modifications. The top heat loss for a single pane air heater was calculated using Equation 3.12.6-1 [44].

$$U_t = \frac{1}{\frac{1}{h_w + h_{rT_p - T_{amb}}} + \frac{1}{h_{rT_p - T_g}} + \frac{L_p}{k_p}}$$ Eq 3.12.6-1

Where $h_{hrTp\text{-}Tamb}$ and $h_{rTp\text{-}Tg}$ are calculated from Equation 3.12.6-2 which is the radiation coefficient [44].

$$h_{r1-2} = \frac{\sigma\left(T_1^2 + T_2^2\right)\left(T_1 + T_2\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$ Eq 3.12.6-2

Where T_1 and T_2 are the two object temperatures (K), ε_1 and ε_2 are the emissivity of the two objects and σ is the Boltzmann constant.

The bottom and side heat loss coefficients are calculated using equations 3.12.4-6 and 3.12.4-7 found in Section 3.12.4. The only other equation that is different from Section 3.12.4 is the collector efficiency, F'. Equation 3.12.6-3 was implemented in the calculation [44].

$$F' = \frac{1}{1 + \frac{U_L}{h_1 + \frac{1}{\frac{1}{h_2} + \frac{1}{h_r}}}}$$ Eq 3.12.6-3

Assuming the temperature of the absorber and the bottom of the duct has the same temperature; h_1 and h_2 are the same [27]. h_1 and h_2 are the convective heat transfer from the duct to the airflow. The convection was calculated using Equation 3.12.4-10 from Section 3.12.4. It was also assumed that the duct had a constant temperature which causes Equation 3.12.6-2 to be simplified to Equation 3.12.6-4.

$$h_r = \frac{\sigma T_p^3}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$ Eq 3.12.6-4

Where T_p is the temperature of the absorber plate.