MSE 5760 Vehicle Battery Cells and Systems

Final Project

Battery Design to Meet DOE Freedom Car Goals

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Parameters:

Table 1 List of Fixed Parameters

Parameter	Value
K	4.11 x 10^{-4} S/cm
σ	0.003 S/cm
C_{dl}	10 $\mu F/cm^2$
$\alpha_a + \alpha_c$	n
n	1
c_i	0.001 mol/cm^3
t^0	0.537
R	96485 s A/mol
F	8.314 J

Table 2 List of Design Paramters

Parameter	Value
l_p	0.005 cm
ε	0.15
ε_{inert}	0.09
R_s	0.0001 cm
Separator Area	9000 cm^2
V_{max}	3.7 V
V_{min}	2.9 V
M	56
SOC_{max}	0.5758
SOC_{min}	0.3737

Table 3 Goals to be met

Parameter	Units	Value
Pulse Discharge Power (10s)	kW	50
Peak regenerative pulse (10s)	kW	40
Total available energy (over DOD ranges where power goals are met)	kWh	1.0
Cold Cranking Power @ -30°C (three 2 sec pulses, 10 sec rest between)	kW	8
Cycle life for specified SOC increments	Cycles	3500
Maximum Volume	Liters	45
Operating voltage limits of battery	Vdc	max≤400 min≥(0.55XVmax)

Mathematical Expressions used for design dependent parameters:

$$i_0 = 121 \exp\left(-\frac{4000}{T}\right) A/cm^2 \ldots\ldots\ldots (1)$$

$$D = 7 \exp\left(-\frac{6123}{T}\right) cm^2/s \ldots\ldots\ldots (2)$$

$$a = \frac{3}{R_s} \cdot (1 - \varepsilon - \varepsilon_{inert}) \cdot \varepsilon \ldots\ldots\ldots (3)$$

$$Area\ of\ Active\ material = a \cdot 2 \cdot l_p \cdot Separator\ Area \ldots\ldots\ldots (4)$$

$$K_{eff} = \varepsilon^{1.5} \cdot K \ldots\ldots\ldots (5)$$

$$\sigma_{eff} = (1 - \varepsilon)^{1.5}\, \sigma \ldots\ldots\ldots (6)$$

$$\nu^2 \qquad \frac{a i_0 F n}{RT}\left(\frac{1}{\sigma_{\mathrm{eff}}} + \frac{1}{\kappa_{\mathrm{eff}}}\right) l_{\mathrm{p}}^2 \qquad \ldots\ldots\ldots (7)$$

$$B_1 \qquad \left(\frac{F^2 \varepsilon c_{\mathrm{i}}}{RT(1 - t_+^0)}\right)\frac{1}{a C_{\mathrm{dl}}} \qquad \ldots\ldots\ldots (8)$$

$$B_2 \qquad \left(\frac{F^2 \varepsilon c_{\mathrm{i}}}{RT(1 - t_+^0)}\right) D_{\mathrm{eff}}\left(\frac{1}{\kappa_{\mathrm{eff}}} + \frac{1}{\sigma_{\mathrm{eff}}}\right) \qquad \ldots\ldots\ldots (9)$$

$$B_3 \qquad \frac{\nu^2}{B_2} \qquad \ldots\ldots\ldots (10)$$

$$D = \varepsilon^{0.5} D \text{ ... ... } (11)$$

$$R_\Omega = \frac{2}{\left(1 + \frac{\kappa_{eff}}{\sigma_{eff}}\right)} \times \left[\frac{\sigma_{.eff}}{l_{.p}} \cdot \left(\frac{1}{2} + 5 \cdot \varepsilon_{.inert}\right)\right]^{-1} \text{ ... ... } (12)$$

$$R_{ct} = \left[2 \frac{\kappa_{eff}}{(\kappa_{eff} + \sigma_{eff})} \frac{\tanh(\nu)}{\nu} + 2 \frac{\sigma_{eff}}{(\kappa_{eff} + \sigma_{eff})}\right] \times \left(\frac{\sigma_{.eff}}{l_{.p}}\right)^{-1} \text{ ... ... } (13)$$

$$R_p = \left[\frac{2}{1 + \frac{\kappa_{eff}}{\sigma_{eff}}} + 2 \frac{\kappa_{eff}}{\kappa_{eff} + \sigma_{eff}} \left[\frac{2t^0}{(2t^0 + B_2)} + \frac{B_2}{(2t^0 + B_2)} \frac{\tanh\left(\sqrt{\frac{\nu^2}{B_2}(2t^0 + B_2)}\right)}{\sqrt{\frac{\nu^2}{B_2}(2t^0 + B_2)}}\right]\right] \times \left(\frac{\sigma_{.eff}}{l_{.p}}\right)^{-1} \text{ ... ... } (14)$$

Internal Aging Resistance:

$$R_{CellA} = R_\Omega + (R_{ct} + R_p) \cdot \exp\left[\frac{\left[\frac{CN}{300} \cdot \frac{(V_{max} - V_{min})}{V}\right]^2}{15 \cdot \left(\frac{AreaOfActiveMaterial}{cm^2}\right)^{\frac{1}{4}}}\right] \text{ ... ... } (15)$$

This V is just the unit Volt

Theoretical Capacity of $LiCoO_2$ = $LiCoO_2$ bulk density x (1/Molecular Weight) x F / 3600

Theoretical Capacity = 1.0198 Ah

To calculate Maximum Capacity, following equation can be used,

$$\text{Maxcapacity} = \frac{\frac{\text{TheoreticalCapacity}}{\text{cm}^3}\cdot\left[2\cdot l_p\cdot \text{SeperatorArea}\cdot\left(1-\varepsilon-\varepsilon_{inert}\right)\right]}{\exp\left[\left[\left[\frac{\frac{CN}{150}\cdot(V_{max}-V_{min})}{50\cdot V}\right]^4\right]\right]}$$

The battery volume is calculated using:

$$Battery\ Volume = M\ x\ 8\ x\ 10^{-4}\ x\ Separator\ Area \ldots\ldots\ldots (16)$$

Table 4 Design Dependent Parameters

Parameter	Value
i_0	0.0001435 A/cm^2
D	5.94 x 10^{-9}cm^2/s
a	3420 cm^{-1}
Area of Active Material	307800 cm^2
K_{eff}	2.38 x 10^{-05}
σ_{eff}	0.0024
D_{eff}	2.3 x 10^{-9}
R_{ohm}	4.4324 ohm cm^2
R_{ct}	4.2202 ohm cm^2
R_p	0.0815 cm^2
RcellA	9.9369 ohm cm^2
R_{cell}	0.0011 ohm
R_{batt}	0.0618 ohm
R_{batt} Cold Crank	0.0620 ohm
Theoretical Capacity	1.0198 Ah
Maximum Capacity	68.4122 Ah

Method to calculate SOCmin and SOCmax:

The relationship between OCV and SOC is already given as

$$OCV = 4 - (1 - SOC)\ 1.2$$

From lecture notes 23-27,

$$OCV = OCV_{max} - (1 - SOC)\ \Delta V$$

So, by comparing these two equations,

$$OCV_{max} = 4 \text{ V and } \Delta V = -1.2$$

The graph of OCV vs 1-SOC is plotted varying SOC from 0 to 1 in the above equation.

Also, as suggested in lecture 23-27, the OCV for charging and discharging is also plotted. Following code was used to plot the graph in Figure 1. The equation used to plot the graph of charging and discharging is given as :

$$V_{batt} = OCV \ \pm IR_{batt}$$

```
DOD=linspace(0,1,10);  % DOD = 1-SOC
ocv=4-((DOD)*1.2);
Pdischarge=50*10^3; % Watts ( Given )
Pcharge=40*10^3; % Watts ( Given)
vchmax=vmax*M;
vdischmin=vmin*M;
Id=Pdischarge/vdischmin; % Current during discharge
Ic=Pcharge/vchmax;  % Currrent during charge
vbc=ocv+((Ic*Rbatt)/M);
vbdc=ocv-((Id*Rbatt)/M);
figure();
plot(DOD,ocv)
title("OCV vs SOC")
xlabel("1-SOC")
ylabel("OCV")
hold on
plot(DOD,vbc,'LineWidth',2);
hold on
plot(DOD,vbdc,'LineWidth',4);
legend('OCV','Vbatt Charge','Vbatt Discharge')
hold off
```

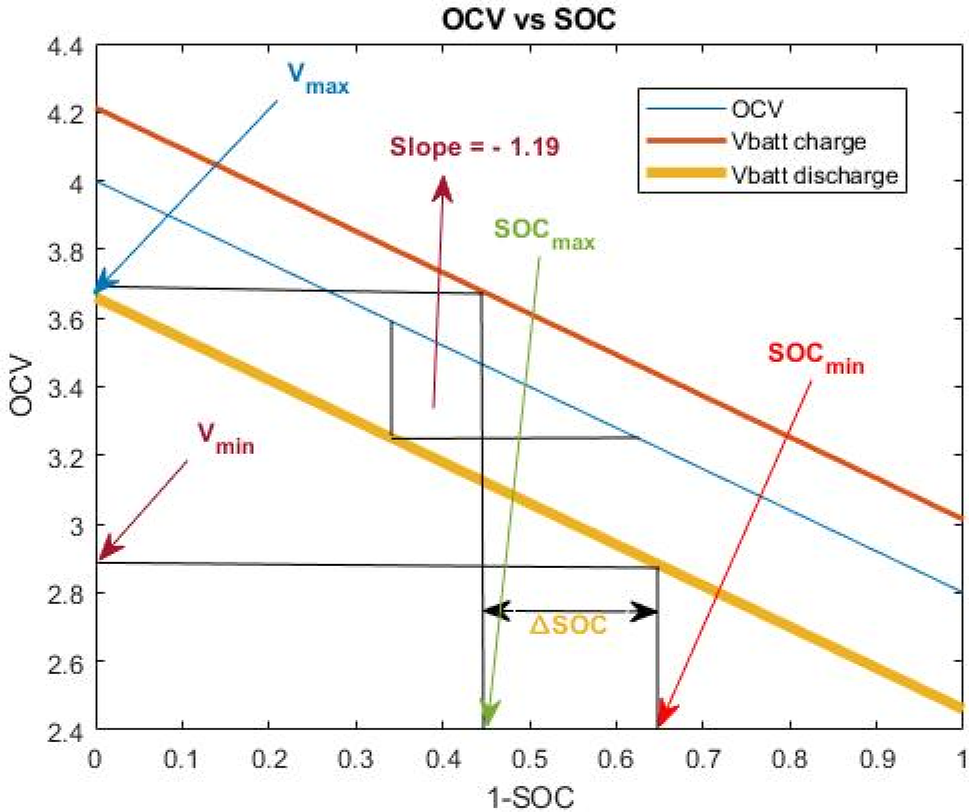


Figure 1

From the graph above, SOC_{min} and SOC_{max} was calculated for V_{min} and V_{max}. The value of $\mathbf{SOC_{max}}$ is **0.5758** and the value of $\mathbf{SOC_{min}}$ is **0.3737.**

PART – A

Q-1,2 Showing peak power requirements satisfied at 3500 cycles using HPPC for discharge at minimum SOC and for charge at maximum SOC.

Answer: As shown in Figure-2, the graph of HPPC for charge and discharge is plotted using the following equation from the lecture 23-27. The figure shows how the peak power requirements are achieved at SOC limits.

$$HPPC(SOC, Area) := \frac{\left[OCV_{Max} + M\cdot(1 - SOC)\right] - V_{MIN}}{\frac{R_{CellA}}{Area}} \cdot V_{MIN}$$

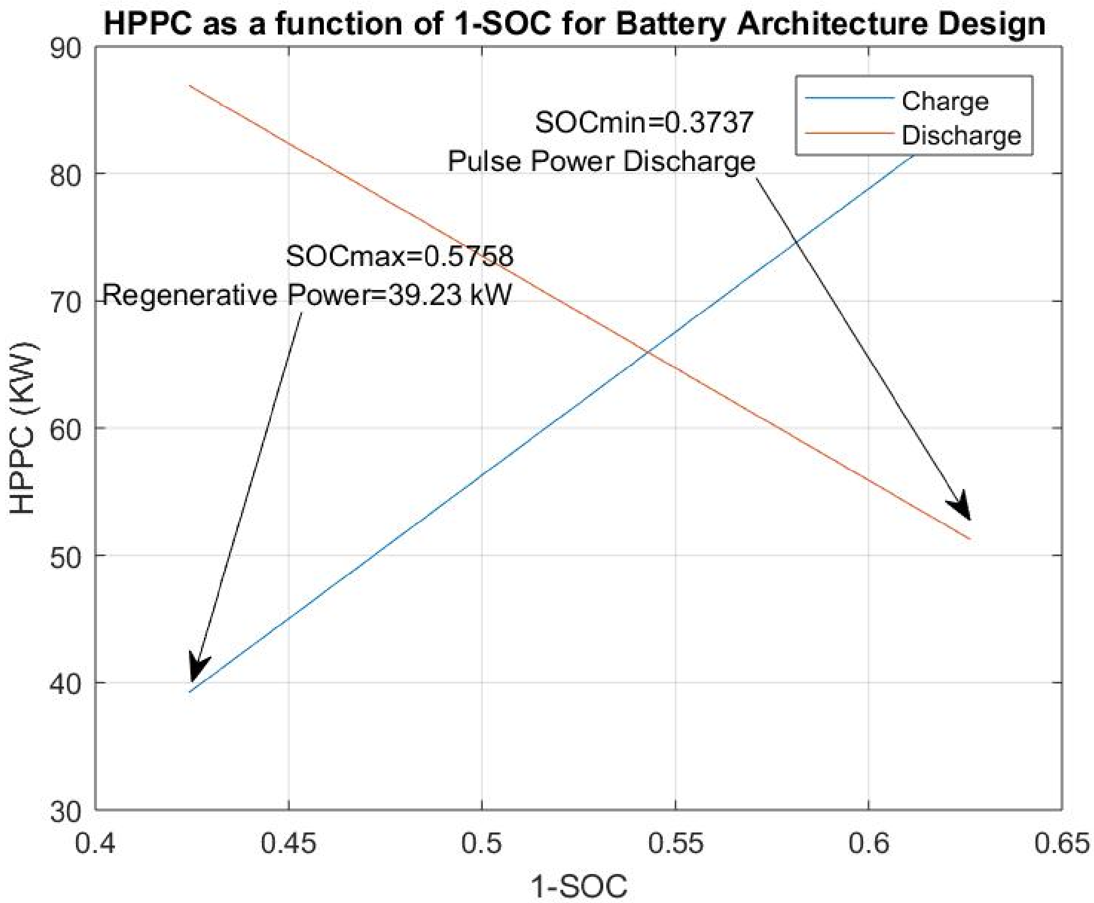


Figure 2 Peak Power Demand Goal

As seen in the figure 2, the peak power for charge at maximum SOC is 39.23 kW which is achieved when compared to 40kW demand of DOE. Whereas the peak power for discharge at minimum at SOC is 51.25 kW which meets 50 kW requirement of DOE. The code used is given below.

```
soc=linspace(SOCmin,SOCmax); % Varying SOC from SOCmin to SOCmax
DOD_B3=1-soc;
ocv_B3=4-(DOD_B3*1.2);
HPPC_charge=((vmax-ocv_B3)*vmax*seperatorarea)/RcellA; % HPPC at
Charge for SOC limits
HPPC_discharge=((ocv_B3-vmin)*vmin*seperatorarea)/RcellA; % HPPC
at Discharge for SOC limits
HPPCcbatt=(HPPC_charge*M)/1000; % HPPC at Charge for SOC limits
for battery architecture
HPPCdbatt=(HPPC_discharge*M)/1000; %% HPPC at discharge for SOC
limits for battery architecture
figure()
plot(DOD_B3,HPPCcbatt,DOD_B3,HPPCdbatt)
grid on
```

```
title ('HPPC as a function of 1-SOC for Battery Architecture
Design ');
xlabel('1-SOC');
ylabel('HPPC (KW)');
legend('Charge','Discharge');
```

Q-3 (a,b,c) Determine C1 rate at 3500 cycles, average battery voltage of discharge for your battery at C1, report the available energy and compare the goal #3 of DOE:

The C1 rate is calculated using the SOC difference that is given by:

$$\Delta SOC = \frac{V_{MIN} - V_{MAX} + \frac{P_{MAX} \cdot RcellA}{N \cdot Area_{cell}} \cdot \left(\frac{1}{V_{MIN}} + \frac{1}{V_{MAX}} \right)}{B}$$

Note: B is the slope here seen in figure 1, which is “deltav” in MATLAB code.

Calculating ΔSOC using above equation, we get the value of 0.1627

Since the cells are to be considered in series the current for the battery and for individual cell will remain same. So, the C1 rate of Battery and C1 rate of cell should remain same. The C1 rate is given by:

$$C1\ rate = \frac{OCV_{max} - V_{min}}{R_{cell} - \left(\frac{Slope}{\Delta SOC - Maximum\ Capacity} \right)}$$

The C1 rate was calculated to be 10.1847.

$OCV_{chargemax}$ and $OCV_{dischargemin}$ are calculated using the following equation:

$$OCV_{charge-discharge} = V_{batt} \mp IR_{batt}$$

Using this we can calculate average battery voltage at C1 rate at 3500 cycles using the following equation:

$$Average\ Battery\ Voltage = \frac{OCV_{charge} + OCV_{discharge}}{2} - C1\ rate\ \times R_{batt}$$

And finally, available energy is calculated using:

$$\text{Energy} = N \cdot \left[\left[Q \cdot Area_{cell} \cdot \Delta SOC \cdot \left(V_{MIN} \right) \right] + \frac{1}{2} \cdot Q \cdot Area_{cell} \cdot \left(|B| \right) \cdot \Delta SOC^{\,2} \right]$$

Where Q x $Area_{cell}$ = Maximum Capacity.

Table 5 C-rate and Energy at 3500 cycles

Parameters	Value
C1 Rate	10.1847
Average Battery Voltage	187.72 V
Available Energy	1.7475 kWh

MATLAB Code:

```
%% Part A Question-3
delta_soc=((vmin-
vmax)+(((Pdischarge*RcellA)/(M*seperatorarea))*(1/vmin+1/vmax)))/deltav;
C1rate=(OCVmax-vmin)/(Rcell-(deltav/((delta_soc)*MaxCap))); % C1 rate
OCVcmax=vchmax-(Ic*Rbatt); % OCV at Charge
OCVdismin=vdischmin+(Id*Rbatt); % OCV at Discharge
Vbattavg=(OCVcmax+OCVdismin)/2-C1rate*Rbatt; % Average Battery Voltage at C1
rate at 3500 cycles
Eavailable=M*((MaxCap.*delta_soc.*vmin)+(0.5.*MaxCap.*deltav.*delta_soc.^2))/
1000 % Available Energy at 3500 cycles
```

Q-4 Cold Crank Test:

Resistance at normal temperature = 0.0618 ohm

Resistance at -30 deg C = 0.0620 ohm

So, there is not much change in resistance seen with change in temperature. Thus, power remains the same at minimum SOC, that is 51.25 kW. This meets the DOE requirement of 8 kW.

The MATLAB code used to calculate the cold crank resistance:

```
%% Part A Question-4
T_CC=243; % K Temperature : -30 deg C
io_CC=121*exp(-4000/T_CC);
D_CC=7*exp(-6123/T_CC);
Deff_CC=(porosity^0.5)*D_CC;
vsquare_CC=((a*io_CC*F*n)/(R*T_CC))*((1/sigmaeff)+(1/Keff))*(lp^2);
v_CC=sqrt(vsquare_CC);
B1_CC=((F^2*porosity*ci)/(R*T_CC*(1-to)))*(1/(a*Cdl));
B2_CC=((F^2*porosity*ci)/(R*T_CC*(1-to)))*Deff_CC*((1/Keff)+(1/sigmaeff));
B3_CC=vsquare_CC/B2_CC;
Rct_CC=(((2*(Keff/(Keff+sigmaeff)))*((tanh(v_CC))/v_CC))+(2*(sigmaeff/(Keff+s
igmaeff))))*(sigmaeff/lp)^-1;
Rp_CC=((2*(Keff/(Keff+sigmaeff)))+(2*(Keff/(Keff+sigmaeff)))*(((2*to)/((2*to)
+B2_CC))+((B2_CC/((2*to)+B2_CC))*((tanh(sqrt((vsquare_CC/B2_CC)*((2*to)+B2_CC
))))/(sqrt((vsquare_CC/B2_CC)*((2*to)+B2_CC)))))))*(sigmaeff/lp)^-1;
RcellA_CC=Rohm+(Rct_CC+Rp_CC)*exp(x/y);
Rcell_CC=RcellA_CC/seperatorarea;
Rbatt_CC=Rcell_CC*M; % Cold Cranking Resistance, it doesn't change much.
```

Q-5: Report Separator area, number of cells in series, battery volume:

Separator Area: 9000 cm^2

M = Number of Cells in series = 56

Using equation (16) to find the battery volume:

Battery Volume = 40.3200 liters which meets DOE goal of 45 liters.

Q-6: Report no. of cells in series and V_{batt} < 400 V according to DOE Goal

M = No. of cells in series = 56

$V_{batt} = M \times V_{max} = 56 \times 3.7 = 207.2 < 400$ V = DOE Goal met

Table 6 Comparison of DOE Goals with the Model Results

Parameter	Units	Value	Model Values
Pulse Discharge Power (10s)	kW	50	51.25
Peak regenerative pulse (10s)	kW	40	39.23
Total available energy (over DOD ranges where power goals are met)	kWh	1.0	1.74
Cold Cranking Power @ -30°C (three 2 sec pulses, 10 sec rest between)	kW	8	51.25
Cycle life for specified SOC increments	Cycles	3500	3500
Maximum Volume	Liters	45	40.32
Operating voltage limits of battery	Vdc	max≤400 min≥(0.55XVmax)	207.2 162.4>(0.55x207.2)

So, as seen in the above table all the goals mentioned for DOE freedom car are met by this battery model.

PART – B

Q-1 : Rcell at CN=0 as a function of separator area:

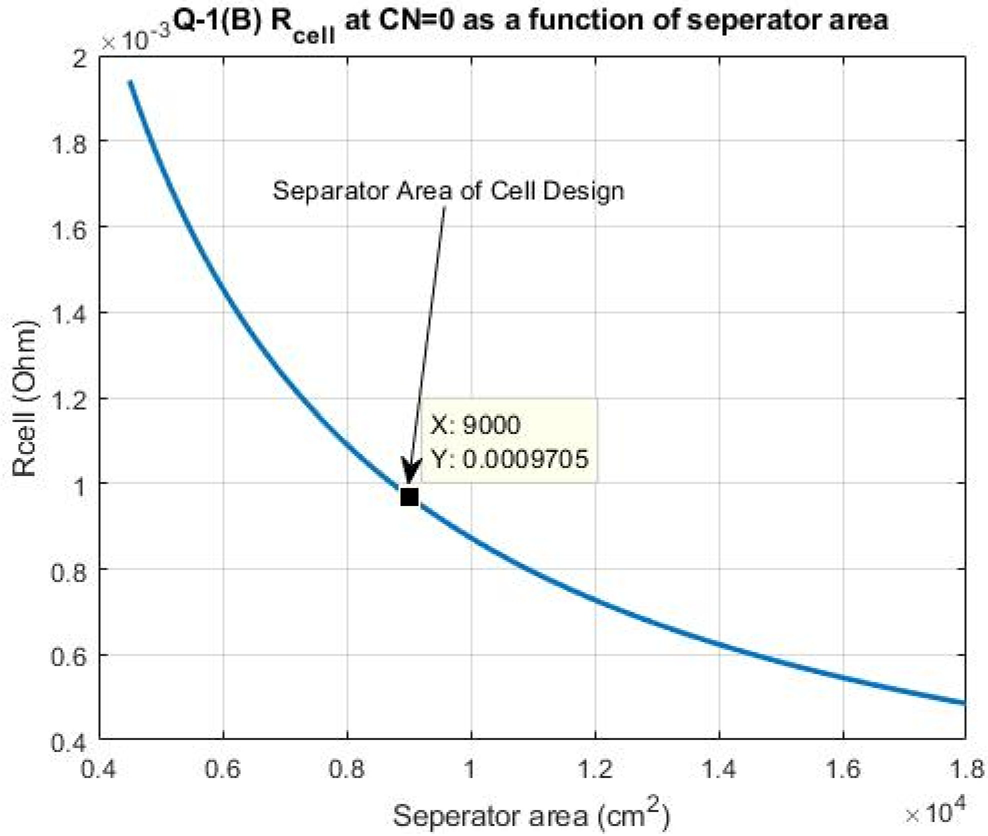


Figure 3 Variable Resistance at CN=0 with separator area

It can be seen that the cell resistance decreases with increase in area, and as a result of battery volume we are unable to use very high separator areas.

```
%% Part B Question-1
cycle1=0; % Cycle Number 0 as required
% Seperator area should be ranging 1/2 times the original seperator area to
% 2 times original seperator area
seperatorarea1=linspace(0.5*seperatorarea,2*seperatorarea,100);
% Area of active material changes as we change the seperator area.
aam1=a*2*(lp*seperatorarea1);
x1=((cycle1/300)*(vmax-vmin))^2;
y1=15*(aam1).^0.25;
RcellA1=Rohm+(Rct+Rp)*exp(x1./y1);
Rcell_CN_0=RcellA1./seperatorarea1;  % Cell Resistance Varying with increase
in separator area
figure()
plot(seperatorarea1,Rcell_CN_0,'linewidth',2 )
grid on
title('Q-1(B) R_c_e_l_l at CN=0 as a function of seperator area');
xlabel('Seperator area (cm^2)');
ylabel('Rcell');
```

Q-2 Cell resistance as a function of cycle number:

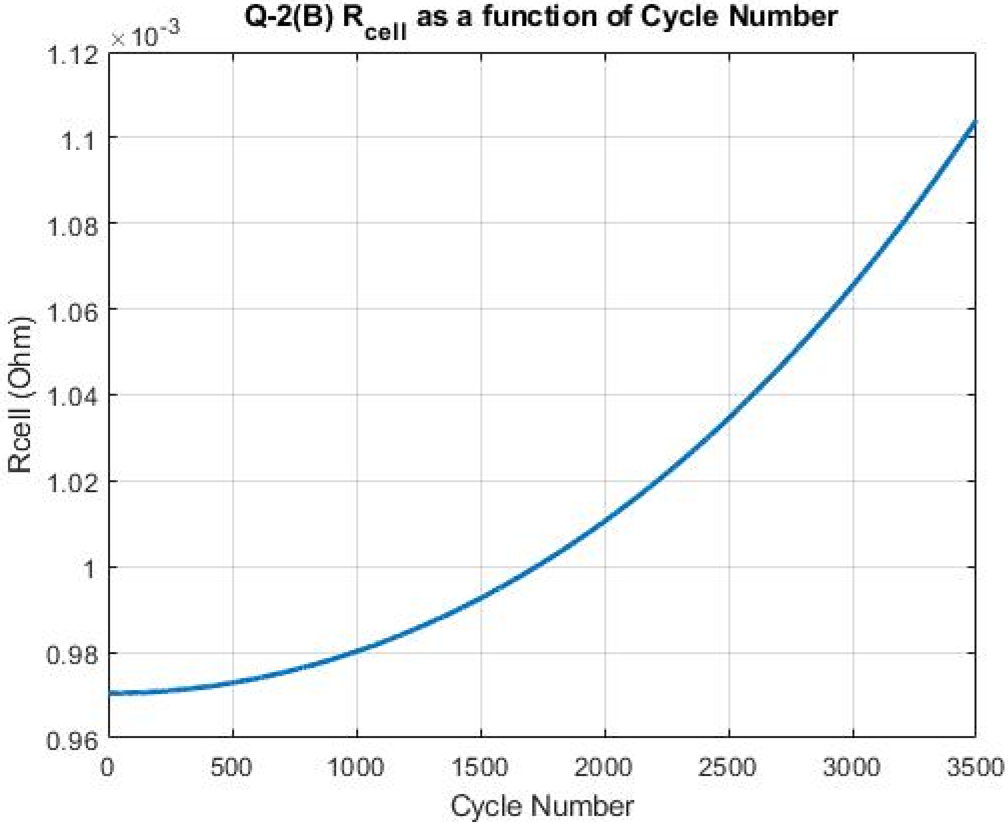


Figure 4 Rcell as a function of cycle number

As seen in the figure above, with the increase in charge-discharge cycles, the resistance of the cell also increases.

```
%% Part B Question-2
cycle2=[0:1:3500]; % Varying Cycle Number
x2=((cycle2/300)*(vmax-vmin)).^2;
y2=15*(aam)^0.25;
RcellA2=Rohm+(Rct+Rp)*exp(x2./y2);
Rcell_CN_0_3500=RcellA2/seperatorarea;  % Cell Resistance Varying with Cycle
= 0 to 3500
figure()
plot(cycle2,Rcell_CN_0_3500,'linewidth',2 )
grid on
title('Q-2(B) R_c_e_l_l as a function of Cycle Number');
xlabel('Cycle Number');
ylabel('Rcell');
```

Q-3 Cell HPPC at SOC= 0.5 at CN =3500 as a function of temperature.

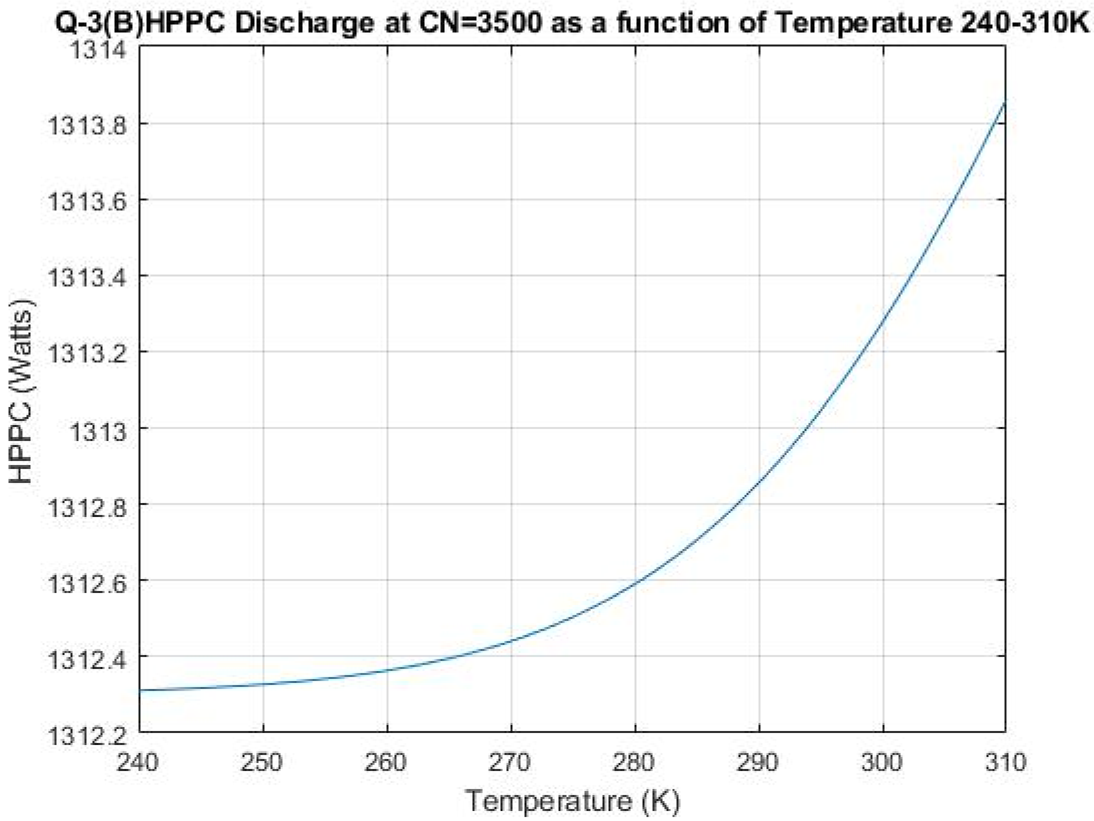


Figure 5 HPPC Variation at SOC=0.5 with Temperature

```
%% Part B Question-3
T_hppc=linspace(240,310); % K  Varying Temperature, so there will be change
in design dependent parameters
io_hppc=121*exp(-4000./T_hppc);
D_hppc=7*exp(-6123./T_hppc);
Deff_hppc=(porosity^0.5).*D_hppc;
vsquare_hppc=((a.*io_hppc*F*n)/(R.*T_hppc))*((1/sigmaeff)+(1/Keff))*(lp^2);
v_hppc=sqrt(vsquare_hppc);
B1_hppc=((F^2*porosity*ci)./(R.*T_hppc*(1-to)))*(1/(a*Cdl));
B2_hppc=((F^2*porosity*ci)./(R.*T_hppc*(1-
to))).*Deff_hppc*((1/Keff)+(1/sigmaeff));
B3_hppc=vsquare_hppc./B2_hppc;
Rct_hppc=(((2*(Keff/(Keff+sigmaeff)))*((tanh(v_hppc))/v_hppc))+(2*(sigmaeff/(
Keff+sigmaeff))))*(sigmaeff/lp)^-1;
Rp_hppc=((2.*(Keff./(Keff+sigmaeff)))+(2.*(Keff./(Keff+sigmaeff))).*(((2.*to)
./((2.*to)+B2_hppc))+((B2_hppc./((2.*to)+B2_hppc)).*((tanh(sqrt((vsquare_hppc
./B2_hppc).*((2.*to)+B2_hppc))))./(sqrt((vsquare_hppc./B2_hppc).*((2.*to)+B2_
hppc)))))))).*(sigmaeff./lp).^-1;
RcellA_hppc=Rohm+(Rct_hppc+Rp_hppc)*exp(x/y);
DOD_hppc=0.5;  % At SOC=0.5, at CN=3500
ocv_hppc=4-((DOD_hppc)*1.2);
```

```
HPPCdischarge=(ocv_hppc-vmin)*vmin*seperatorarea./RcellA_hppc;
figure()
plot(T_hppc,HPPCdischarge)
grid on
title('Q-3(B)HPPC Discharge at CN=3500 as a function of Temperature 240-
310K');
xlabel('Temperature');
ylabel('HPPC (Watts)') ;
```

Q-4 HPPC cell for chosen vmin and vmax as a function of 1-SOC

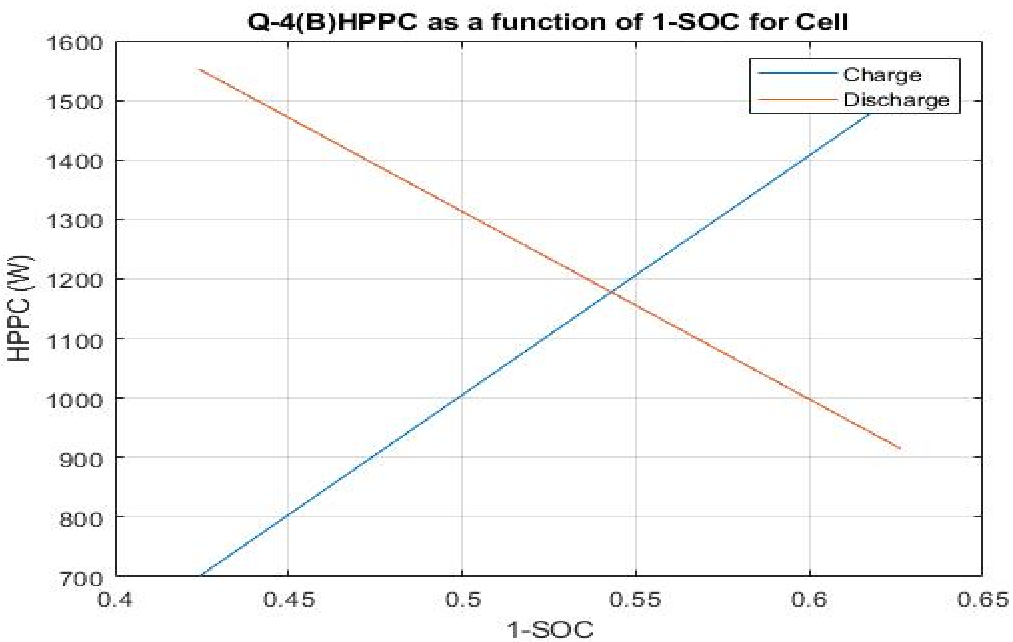


Figure 6 HPPC Cell as Chosen Vmin and Vmax

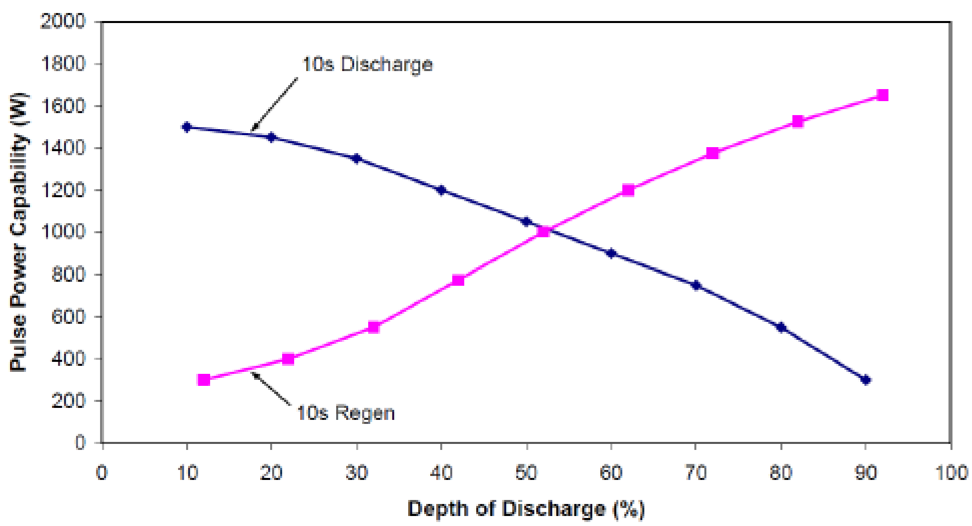


Figure-6 resembles the DOE curve for the same HPPC vs DOD plot.

As the DOD increases less discharge power is available, whereas more power is available for charging the battery when DOD increases.

```
%% Part B Question-4
soc=linspace(SOCmin,SOCmax); % Varying SOC from SOCmin to SOCmax as we need
to plot is at chosen value of Vmin and Vmax
DOD_B3=1-soc;
ocv_B3=4-(DOD_B3*1.2);
HPPC_charge=((vmax-ocv_B3)*vmax*seperatorarea)/RcellA; % HPPC at Charge for
SOC limits
HPPC_discharge=((ocv_B3-vmin)*vmin*seperatorarea)/RcellA; % HPPC at Discharge
for SOC limits
figure()
plot(DOD_B3,HPPC_charge,DOD_B3,HPPC_discharge)
grid on
title ("Q-4(B)HPPC as a function of 1-SOC for Cell");
xlabel("1-SOC");
ylabel("HPPC (W)");
legend("Charge","Discharge");
```

Q-5 Repeat 2-4 for the battery Architecture

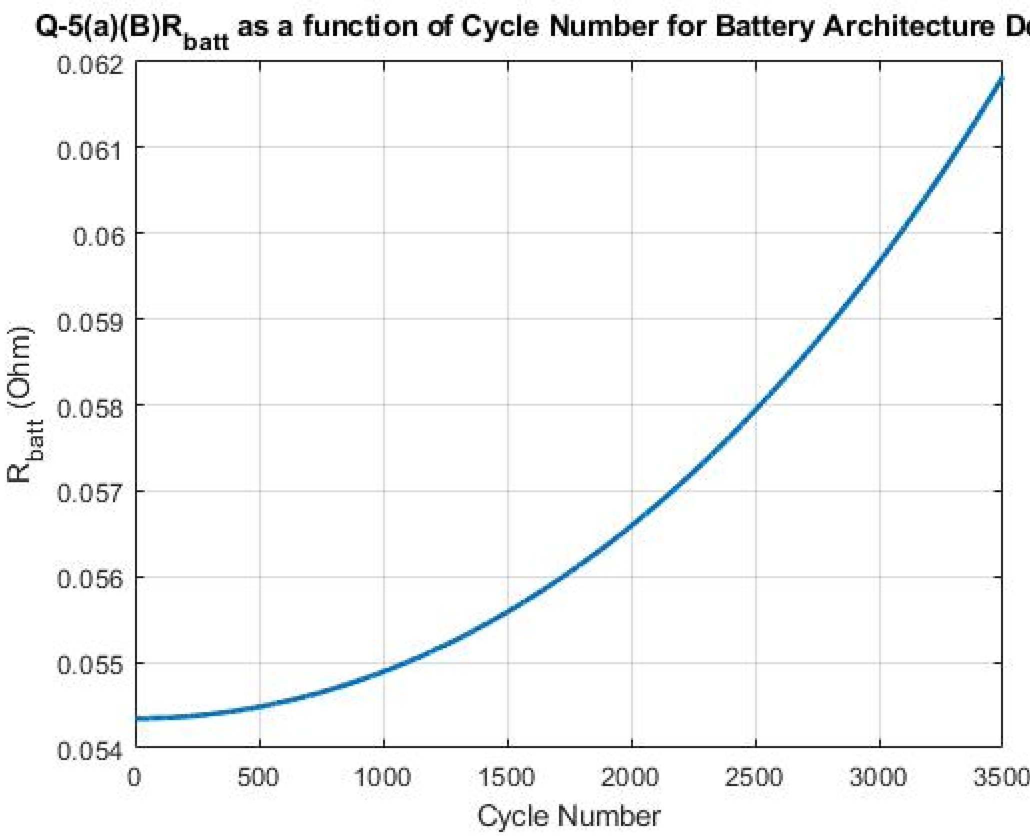


Figure 7 Rbatt as a Function of CN for the Battery Architecture

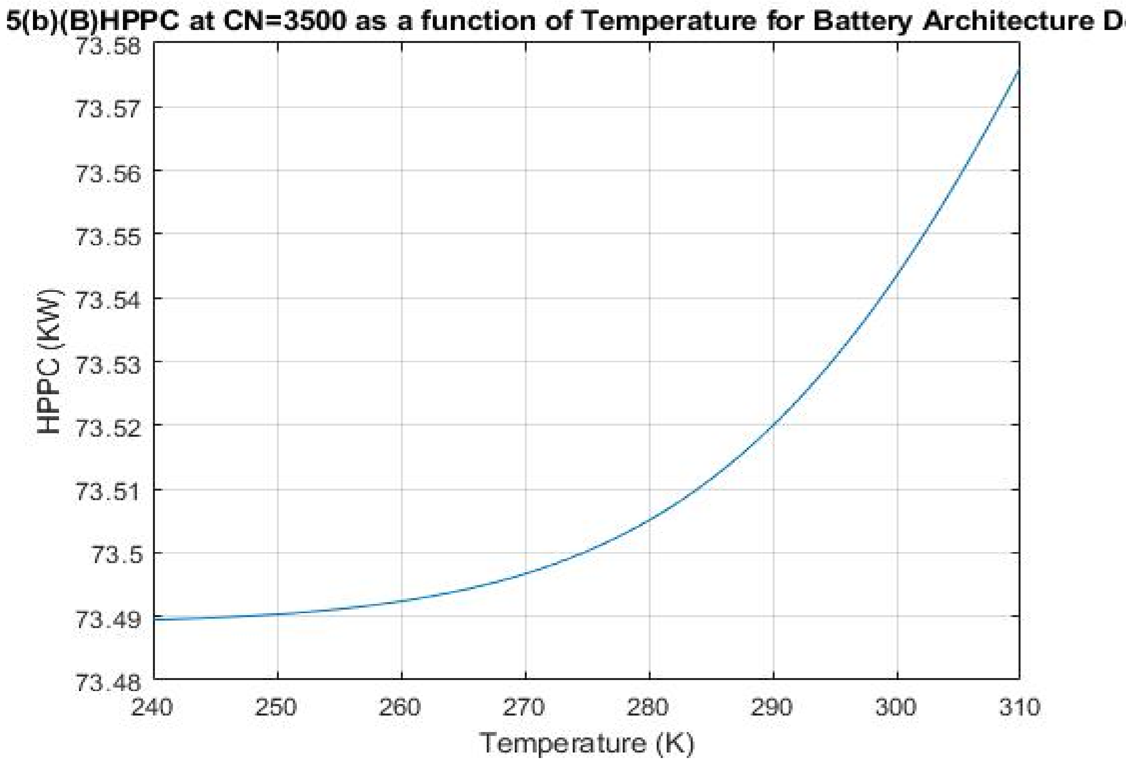


Figure 8 HPPC at CN=3500 for varying temperature

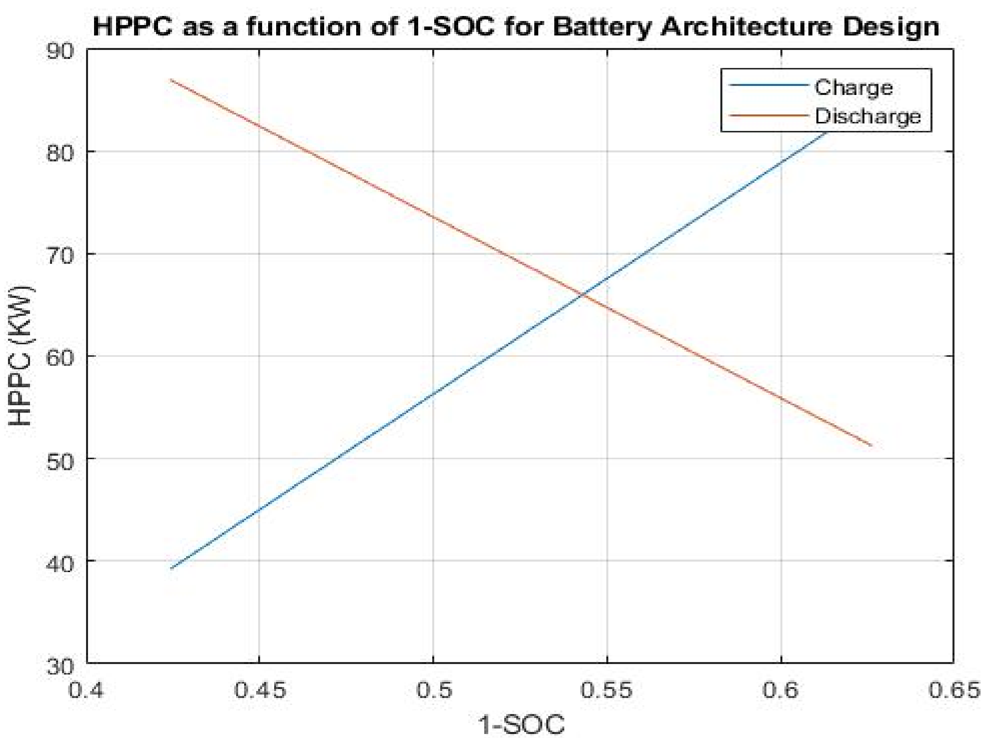


Figure 9 HPPC for Vmin and Vmax at

Figures 7, 8 and 9 shows the same plots as in questions 2, 3, 4. The only differences that the latter ones are for cell, while figures 7, 8, 9 shows the same for battery design.

```
%% Part B Question-5
%2 for Battery Architecture Design
x5=((cycle2/300)*((vmax)-(vmin))).^2;
y5=15*(aam)^0.25;
RcellA3=Rohm+(Rct+Rp)*exp(x5./y5);
Rcell_CN5_0_3500=RcellA3/seperatorarea;
Rbatt_5=Rcell_CN5_0_3500*M;
figure()
plot(cycle2,Rbatt_5,'linewidth',2)
grid on
title('Q-5(a)(B)R_b_a_t_t as a function of Cycle Number for Battery
Architecture Design');
xlabel('Cycle Number');
ylabel('R_b_a_t_t (Ohm)');

%3 for Battery Architecture Design
HPPCdbatt=(HPPCdischarge*M)/1000;
figure()
plot(T_hppc,HPPCdbatt)
grid on
title('Q-5(b)(B)HPPC at CN=3500 as a function of Temperature for Battery
Architecture Design')
xlabel('Temperature (K)');
ylabel('HPPC (KW)');

%4 for Battery Architecture Design
HPPCcbatt=(HPPC_charge*M)/1000; % HPPC at Charge for SOC limits for battery
architecture
HPPCdbatt=(HPPC_discharge*M)/1000; %% HPPC at discharge for SOC limits for
battery architecture
figure()
plot(DOD_B3,HPPCcbatt,DOD_B3,HPPCdbatt)
grid on
title ('HPPC as a function of 1-SOC for Battery Architecture Design ');
xlabel('1-SOC');
ylabel('HPPC (KW)');
legend('Charge','Discharge');
```

Q-6 Delta SOC as a function of battery power:

Here, as we see in figure-10 the battery power is maximum when ΔSOC goes to 0. The matches with the Newman's theory where the peak power is observed at ΔSOC = 0. And the power reduces to zero with increase in ΔSOC.

```
%% Part B Question-6
Pmax_6_3500=(((vmax*vmin)*(vmax-vmin))/(Rcell*(vmax+vmin)))*M/1000;  % Max
power at 3500 cycles
CN_6_0=0; % Calculation of power at 0 cycles
x_6=((CN_6_0/300)*(vmax-vmin))^2;
y_6=15*(aam)^0.25;
RcellA_6=Rohm+(Rct+Rp)*exp(x_6/y_6);
Rcell_6=RcellA_6/seperatorarea;
Rbatt_6=Rcell_6*M;
```

```
Pmax_6_0=(((vmax*vmin)*(vmax-vmin))/(Rcell_6*(vmax+vmin)))*M /1000; % Max
power at 0 cycles
Pmax_6_3500_a=[0:0.1:Pmax_6_3500];
Pmax_6_0_a=[0:0.1:Pmax_6_0];
deltasoc_3500=1-(Pmax_6_3500_a/Pmax_6_3500);  % delta SOC at 3500 cycles
deltasoc_0=1-(Pmax_6_0_a/Pmax_6_0); % delta SOC at 0 cycles
figure()
plot(Pmax_6_3500_a,deltasoc_3500,Pmax_6_0_a,deltasoc_0)
grid on
title('Delta SOC as a function of Battery Power');
xlabel('Battery Power (kW)');
ylabel('Delta SOC');
legend('Cycle Number=3500','Cycle Number=0');
```

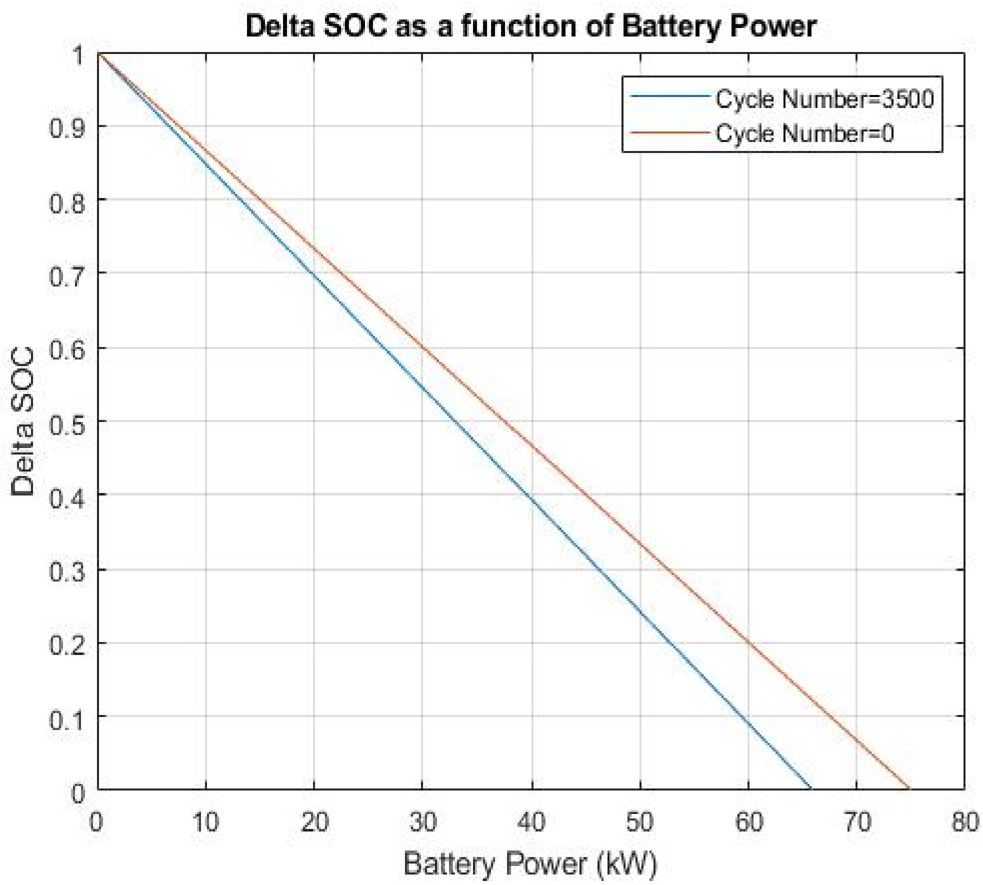


Figure 10 Delta SOC as function of Battery

Q-7 Delta SOC as function of Battery power at 240 K

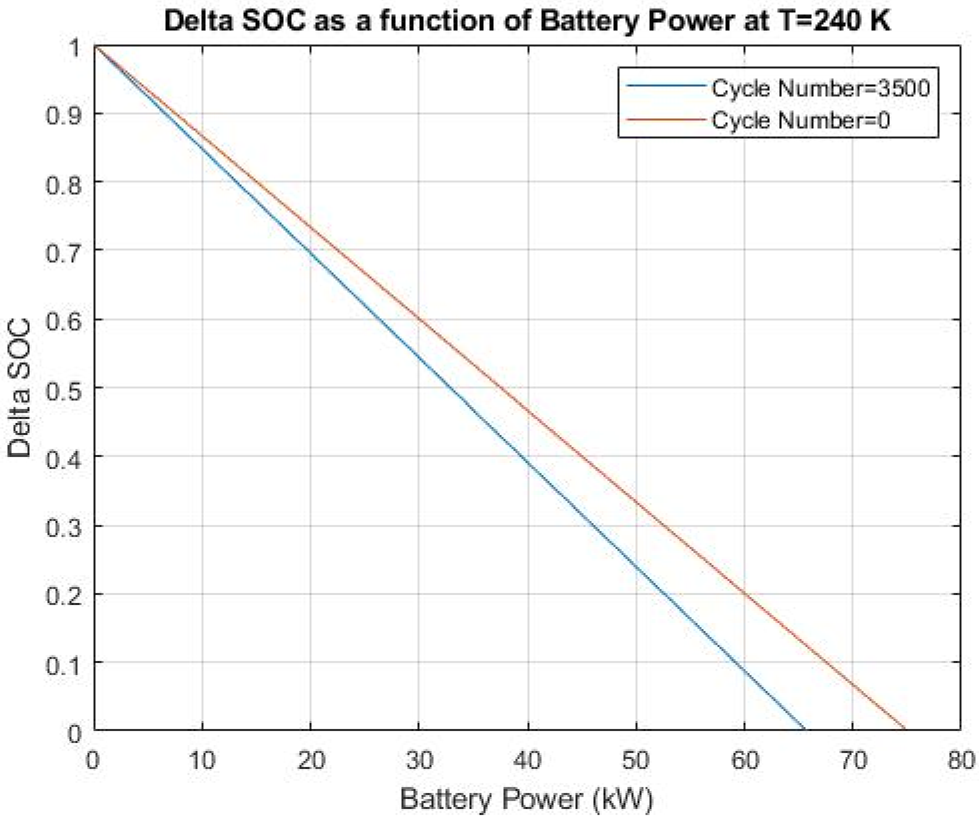


Figure 11 Delta SOC as function of Battery power at 240 K

Figure 11 shows the same graph as Figure 10, as we discussed earlier the resistance is not changing much so the battery power will remain the same and therefore ΔSOC also remains same.

```
%% Part B Question-7
T_240=240; % K  Changing temperature changes design dependent variables
% Calculating new parameters with temperature change
io_240=121*exp(-4000/T_240);
D_240=7*exp(-6123/T_240);
Deff_240=(porosity^0.5)*D_240;
vsquare_240=((a*io_240*F*n)/(R*T_240))*((1/sigmaeff)+(1/Keff))*(lp^2);
v_240=sqrt(vsquare_240);
B1_240=((F^2*porosity*ci)/(R*T_240*(1-to)))*(1/(a*Cdl));
B2_240=((F^2*porosity*ci)/(R*T_240*(1-to)))*Deff_240*((1/Keff)+(1/sigmaeff));
B3_240=vsquare_240/B2_240;
Rct_240=(((2*(Keff/(Keff+sigmaeff)))*((tanh(v_240))/v_240))+(2*(sigmaeff/(Kef
f+sigmaeff))))*(sigmaeff/lp)^-1;
Rp_240=((2*(Keff/(Keff+sigmaeff)))+(2*(Keff/(Keff+sigmaeff)))*(((2*to)/((2*to
)+B2_240))+((B2_240/((2*to)+B2_240))*((tanh(sqrt((vsquare_240/B2_240)*((2*to)
+B2_240))))/(sqrt((vsquare_240/B2_240)*((2*to)+B2_240)))))))*(sigmaeff/lp)^-
1;
RcellA_240=Rohm+(Rct_240+Rp_240)*exp(x/y);
Rcell_240=RcellA_240/seperatorarea;
```

```
Rbatt_240=Rcell_240*M;
Pmax_7_3500=(((vmax*vmin)*(vmax-vmin))/(Rcell_240*(vmax+vmin)))*M/1000;  %
Max power at 3500 cycles
RcellA_7=Rohm+(Rct_240+Rp_240)*exp(x_6/y_6);
Rcell_7=RcellA_7/seperatorarea;
Rbatt_7=Rcell_7*M;
Pmax_7_0=(((vmax*vmin)*(vmax-vmin))/(Rcell_7*(vmax+vmin)))*M/1000; % Max
power at 0 cycles
Pmax_7_3500_a=[0:0.1:Pmax_7_3500];
Pmax_7_0_a=[0:0.1:Pmax_7_0];
deltasoc_3500_7=1-(Pmax_7_3500_a/Pmax_7_3500); % delta SOC at 3500 cycles
deltasoc_0_7=1-(Pmax_7_0_a/Pmax_6_0);% delta SOC at 0 cycles
figure(9)
plot(Pmax_7_3500_a,deltasoc_3500_7,Pmax_7_0_a,deltasoc_0_7)
grid on
title('Delta SOC as a function of Battery Power at T=240 K');
xlabel('Battery Power (kW)');
ylabel('Delta SOC');
legend('Cycle Number=3500','Cycle Number=0');
```

Q-8 Usable Capacity at 25kW as a function of CN= 0 to 3500

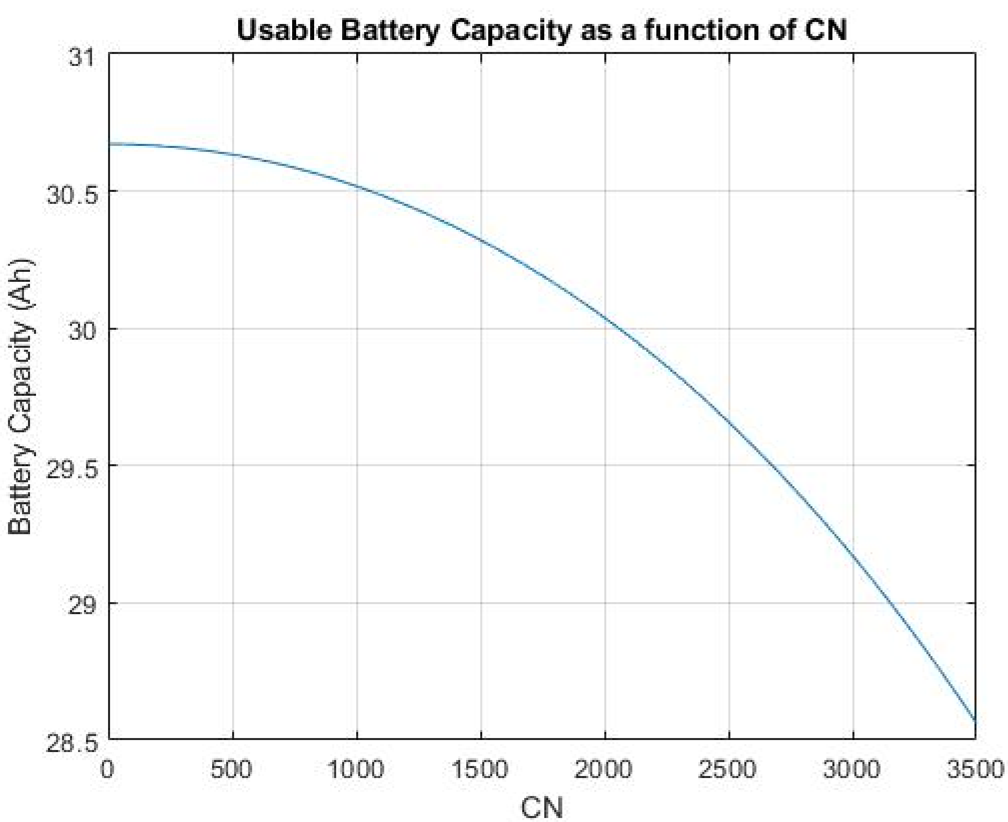


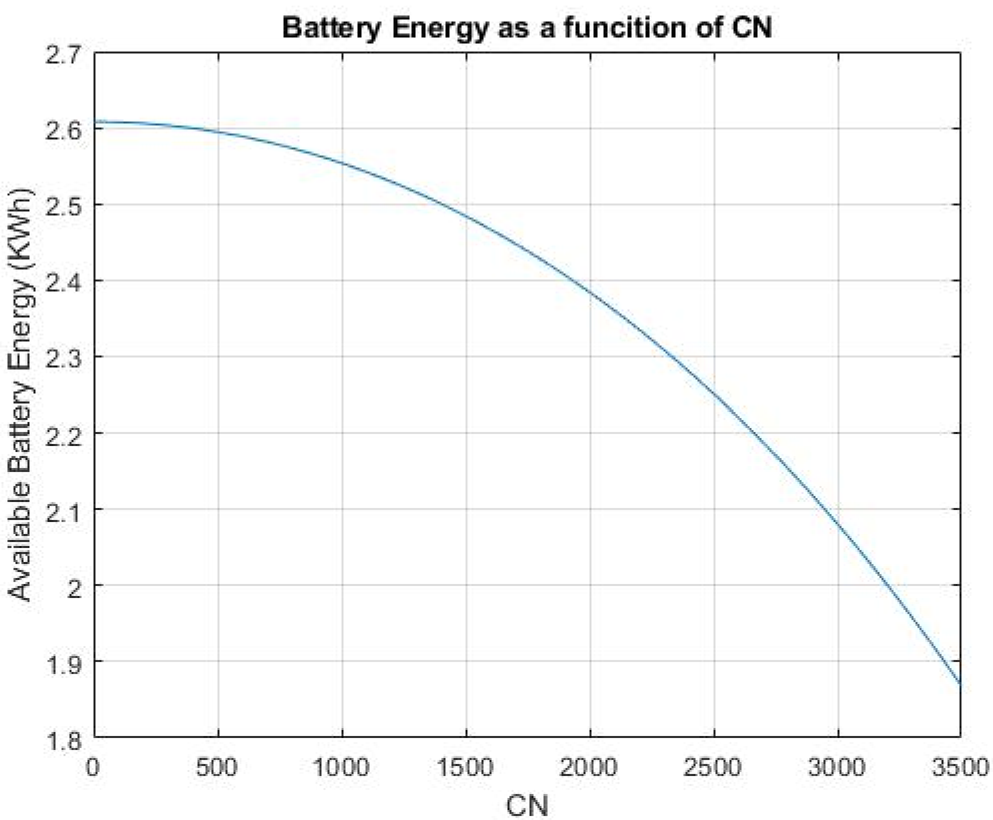
Figure 12 Usable Capacity with Cycle Number

As seen in Figure 12 the usable capacity decreases with increase in the number of charge discharge cycles.

```
%% Part B Question-8

P_max_B8=25*10^3;
delta_soc_B8=((vmin-
vmax)+(((P_max_B8*RcellA3)/(M*seperatorarea))*(1/vmin+1/vmax)))/deltav; %
Delta SOC at 25 kW for 0 to 3500 cycles
batt_cap=MaxCap*abs(delta_soc_B8); % Usable Capacity of Battery
figure()
plot(cycle2,batt_cap)
grid on
title ('Usable Battery Capacity as a function of CN');
xlabel('CN');
ylabel('Battery Capacity (Ah)');
```

Q-9 Available battery energy at C rate as a function of CN= 0 to 3500




```
%% Part B Question-9

Rbatt_9=Rcell_CN_0_3500*M;
delta_9=((vmin-
vmax)+(((50000*RcellA2)/(M*seperatorarea))*(1/vmin+1/vmax)))/deltav;  % delta
SOC at pulse discharge power for 0 to 3500 cycles
C1rate_9=(OCVmax-vmin)*M./(Rcell_CN_0_3500*M-(deltav*M./((delta_9)*MaxCap)));
Eavailable_9=M*((MaxCap.*delta_9.*vmin)+(0.5.*MaxCap.*abs(deltav).*delta_9.^2
))/1000; % Available energy using C1 rate
```

```
figure()
plot (cycle2,Eavailable_9)
grid on
title ('Battery Energy as a funcition of CN');
xlabel ("CN");
ylabel ('Available Battery Energy (KWh)');
```

Full Length MATLAB CODE :

```
%Fixed Parameters
sigma=0.003; % S/cm
Cdl=10; %uF/sq cm
n=1;
alphaac=n;
ci=0.001; % mol/ cubic cm
to=0.537;
OCVmax=4;
deltav=-1.19;
bdensity=3.7; %g/cubic cm
K=4.11*10^-4; %S/cm
F=96485; % sA/mol
R=8.314;% J
%Design Parameters
T=293.15; % K
lp=0.005; %cm
porosity=0.15;
porosityinert=0.09;
cycle=3500;
vmax=3.7; % V
vmin=2.9; %V
M=56; % No. of cells in series
seperatorarea=9000; % m^2
Rs=0.0001; % cm  1um
% Parameters dependent on design parameters
io=121*exp(-4000/T);
D=7*exp(-6123/T);
a=(3/Rs)*(1-porosity-porosityinert)*porosity; % Active surface area per unit
volume
aam=a*2*(lp*seperatorarea); % Area of active material
Keff=(porosity^1.5)*K;
sigmaeff=(1-porosity)^1.5*sigma;
Deff=(porosity^0.5)*D;
vsquare=((a*io*F*n)/(R*T))*((1/sigmaeff)+(1/Keff))*(lp^2);
v=sqrt(vsquare);
B1=((F^2*porosity*ci)/(R*T*(1-to)))*(1/(a*Cdl));
B2=((F^2*porosity*ci)/(R*T*(1-to)))*Deff*((1/Keff)+(1/sigmaeff));
B3=vsquare/B2;
bvol=M*8*10^-4*seperatorarea*10^-4*10^3; % Battery Volume
Rohm=(2/(1+(Keff/sigmaeff)))*((sigmaeff/lp)*(0.5+(5*porosityinert)))^-1;
Rct=(((2*(Keff/(Keff+sigmaeff)))*((tanh(v))/v))+(2*(sigmaeff/(Keff+sigmaeff))
))*(sigmaeff/lp)^-1;
Rp=((2*(Keff/(Keff+sigmaeff)))+(2*(Keff/(Keff+sigmaeff)))*(((2*to)/((2*to)+B2
))+((B2/((2*to)+B2))*((tanh(sqrt((vsquare/B2)*((2*to)+B2))))/(sqrt((vsquare/B
2)*((2*to)+B2)))))))*(sigmaeff/lp)^-1;
x=((cycle/300)*(vmax-vmin))^2;
y=15*(aam)^0.25;
```

```
RcellA=Rohm+(Rct+Rp)*exp(x/y);
Rcell=RcellA/seperatorarea;
Rbatt=Rcell*M;
Tcap=(F*bdensity)/(97.24*3600);
MaxCap=(Tcap*(2*lp*seperatorarea*(1-porosity-
porosityinert)))/exp((((cycle/150)*(vmax-vmin))/50)^4);
DOD=linspace(0,1,10);  % DOD = 1-SOC
ocv=4-((DOD)*1.2);
Pdischarge=50*10^3; % Watts ( Given )
Pcharge=40*10^3; % Watts ( Given)
vchmax=vmax*M;
vdischmin=vmin*M;
Id=Pdischarge/vdischmin; % Current during discharge
Ic=Pcharge/vchmax;  % Currrent during charge
vbc=ocv+((Ic*Rbatt)/M);
vbdc=ocv-((Id*Rbatt)/M);
figure();
plot(DOD,ocv)
title("OCV vs SOC")
xlabel("1-SOC")
ylabel("OCV")
hold on
plot(DOD,vbc,'LineWidth',2);
hold on
plot(DOD,vbdc,'LineWidth',4);
legend('OCV','Vbatt Charge','Vbatt Discharge')
hold off
SOCmin=0.3737;  % Obtained from the plot above
SOCmax=0.5758;  % Obtained from the plot above
%% Part A Question-3
delta_soc=((vmin-
vmax)+(((Pdischarge*RcellA)/(M*seperatorarea))*(1/vmin+1/vmax)))/deltav;
C1rate=(OCVmax-vmin)/(Rcell-(deltav/((delta_soc)*MaxCap))); % C1 rate
OCVcmax=vchmax-(Ic*Rbatt); % OCV at Charge
OCVdismin=vdischmin+(Id*Rbatt); % OCV at Discharge
Vbattavg=(OCVcmax+OCVdismin)/2-C1rate*Rbatt; % Average Battery Voltage at C1
rate at 3500 cycles
Eavailable=M*((MaxCap.*delta_soc.*vmin)+(0.5.*MaxCap.*deltav.*delta_soc.^2))/
1000 % Available Energy at 3500 cycles
%% Part A Question-4
T_CC=243; % K Temperature : -30 deg C
io_CC=121*exp(-4000/T_CC);
D_CC=7*exp(-6123/T_CC);
Deff_CC=(porosity^0.5)*D_CC;
vsquare_CC=((a*io_CC*F*n)/(R*T_CC))*((1/sigmaeff)+(1/Keff))*(lp^2);
v_CC=sqrt(vsquare_CC);
B1_CC=((F^2*porosity*ci)/(R*T_CC*(1-to)))*(1/(a*Cdl));
B2_CC=((F^2*porosity*ci)/(R*T_CC*(1-to)))*Deff_CC*((1/Keff)+(1/sigmaeff));
B3_CC=vsquare_CC/B2_CC;
Rct_CC=(((2*(Keff/(Keff+sigmaeff)))*((tanh(v_CC))/v_CC))+(2*(sigmaeff/(Keff+s
igmaeff))))*(sigmaeff/lp)^-1;
Rp_CC=((2*(Keff/(Keff+sigmaeff)))+(2*(Keff/(Keff+sigmaeff)))*(((2*to)/((2*to)
+B2_CC))+((B2_CC/((2*to)+B2_CC))*((tanh(sqrt((vsquare_CC/B2_CC)*((2*to)+B2_CC
))))/(sqrt((vsquare_CC/B2_CC)*((2*to)+B2_CC)))))))*(sigmaeff/lp)^-1;
RcellA_CC=Rohm+(Rct_CC+Rp_CC)*exp(x/y);
Rcell_CC=RcellA_CC/seperatorarea;
```

```
Rbatt_CC=Rcell_CC*M; % Cold Cranking Resistance, it doesn't change much, so
power does not changes.
%% Part B Question-1

cycle1=0; % Cycle Number 0 as required
% Seperator area should be ranging 1/2 times the original seperator area to
% 2 times original seperator area
seperatorarea1=linspace(0.5*seperatorarea,2*seperatorarea,100);
% Area of active material changes as we change the seperator area.
aam1=a*2*(lp*seperatorarea1);
x1=((cycle1/300)*(vmax-vmin))^2;
y1=15*(aam1).^0.25;
RcellA1=Rohm+(Rct+Rp)*exp(x1./y1);
Rcell_CN_0=RcellA1./seperatorarea1;  % Cell Resistance Varying with increase
in separator area
figure()
plot(seperatorarea1,Rcell_CN_0,'linewidth',2 )
grid on
title('Q-1(B) R_c_e_l_l at CN=0 as a function of seperator area');
xlabel('Seperator area (cm^2)');
ylabel('Rcell (Ohm)');

%% Part B Question-2
cycle2=linspace(0,3500,3500); % Varying Cycle Number
x2=((cycle2/300)*(vmax-vmin)).^2;
y2=15*(aam)^0.25;
RcellA2=Rohm+(Rct+Rp)*exp(x2./y2);
Rcell_CN_0_3500=RcellA2/seperatorarea;  % Cell Resistance Varying with Cycle
= 0 to 3500
figure()
plot(cycle2,Rcell_CN_0_3500,'linewidth',2 )
grid on
title('Q-2(B) R_c_e_l_l as a function of Cycle Number');
xlabel('Cycle Number');
ylabel('Rcell (Ohm)');
%% Part B Question-3
T_hppc=linspace(240,310); % K  Varying Temperature, so there will be change
in design dependent parameters
io_hppc=121*exp(-4000./T_hppc);
D_hppc=7*exp(-6123./T_hppc);
Deff_hppc=(porosity^0.5).*D_hppc;
vsquare_hppc=((a.*io_hppc*F*n)/(R.*T_hppc))*((1/sigmaeff)+(1/Keff))*(lp^2);
v_hppc=sqrt(vsquare_hppc);
B1_hppc=((F^2*porosity*ci)./(R.*T_hppc*(1-to)))*(1/(a*Cdl));
B2_hppc=((F^2*porosity*ci)./(R.*T_hppc*(1-
to))).*Deff_hppc*((1/Keff)+(1/sigmaeff));
B3_hppc=vsquare_hppc./B2_hppc;
Rct_hppc=(((2*(Keff/(Keff+sigmaeff)))*((tanh(v_hppc))/v_hppc))+(2*(sigmaeff/(
Keff+sigmaeff))))*(sigmaeff/lp)^-1;
Rp_hppc=((2.*(Keff./(Keff+sigmaeff)))+(2.*(Keff./(Keff+sigmaeff))).*(((2.*to)
./((2.*to)+B2_hppc))+((B2_hppc./((2.*to)+B2_hppc)).*((tanh(sqrt((vsquare_hppc
./B2_hppc).*((2.*to)+B2_hppc))))./(sqrt((vsquare_hppc./B2_hppc).*((2.*to)+B2_
hppc))))))).*(sigmaeff./lp).^-1;
RcellA_hppc=Rohm+(Rct_hppc+Rp_hppc)*exp(x/y);
DOD_hppc=0.5;  % At SOC=0.5, at CN=3500
ocv_hppc=4-((DOD_hppc)*1.2);
```

```
HPPCdischarge=(ocv_hppc-vmin)*vmin*seperatorarea./RcellA_hppc;
figure()
plot(T_hppc,HPPCdischarge)
grid on
title('Q-3(B)HPPC Discharge at CN=3500 as a function of Temperature 240-
310K');
xlabel('Temperature (K)');
ylabel('HPPC (Watts)') ;
%% Part B Question-4
soc=linspace(SOCmin,SOCmax); % Varying SOC from SOCmin to SOCmax as we need
to plot is at chosen value of Vmin and Vmax
DOD_B3=1-soc;
ocv_B3=4-(DOD_B3*1.2);
HPPC_charge=((vmax-ocv_B3)*vmax*seperatorarea)/RcellA; % HPPC at Charge for
SOC limits
HPPC_discharge=((ocv_B3-vmin)*vmin*seperatorarea)/RcellA; % HPPC at Discharge
for SOC limits
figure()
plot(DOD_B3,HPPC_charge,DOD_B3,HPPC_discharge)
grid on
title ("Q-4(B)HPPC as a function of 1-SOC for Cell");
xlabel("1-SOC");
ylabel("HPPC (W)");
legend("Charge","Discharge");

%% Part B Question-5
%2 for Battery Architecture Design
x5=((cycle2/300)*((vmax)-(vmin))).^2;
y5=15*(aam)^0.25;
RcellA3=Rohm+(Rct+Rp)*exp(x5./y5);
Rcell_CN5_0_3500=RcellA3/seperatorarea;
Rbatt_5=Rcell_CN5_0_3500*M;
figure()
plot(cycle2,Rbatt_5,'linewidth',2)
grid on
title('Q-5(a)(B)R_b_a_t_t as a function of Cycle Number for Battery
Architecture Design');
xlabel('Cycle Number');
ylabel('R_b_a_t_t (Ohm)');

%3 for Battery Architecture Design
HPPCdbatt=(HPPCdischarge*M)/1000;
figure()
plot(T_hppc,HPPCdbatt)
grid on
title('Q-5(b)(B)HPPC at CN=3500 as a function of Temperature for Battery
Architecture Design')
xlabel('Temperature (K)');
ylabel('HPPC (KW)');

%4 for Battery Architecture Design
HPPCcbatt=(HPPC_charge*M)/1000; % HPPC at Charge for SOC limits for battery
architecture
HPPCdbatt=(HPPC_discharge*M)/1000; %% HPPC at discharge for SOC limits for
battery architecture
figure()
```

```
plot(DOD_B3,HPPCcbatt,DOD_B3,HPPCdbatt)
grid on
title ('HPPC as a function of 1-SOC for Battery Architecture Design ');
xlabel('1-SOC');
ylabel('HPPC (KW)');
legend('Charge','Discharge');

%% Part B Question-6

Pmax_6_3500=(((vmax*vmin)*(vmax-vmin))/(Rcell*(vmax+vmin)))*M/1000;  % Max
power at 3500 cycles
CN_6_0=0; % Calculation of power at 0 cycles
x_6=((CN_6_0/300)*(vmax-vmin))^2;
y_6=15*(aam)^0.25;
RcellA_6=Rohm+(Rct+Rp)*exp(x_6/y_6);
Rcell_6=RcellA_6/seperatorarea;
Rbatt_6=Rcell_6*M;
Pmax_6_0=(((vmax*vmin)*(vmax-vmin))/(Rcell_6*(vmax+vmin)))*M /1000; % Max
power at 0 cycles
Pmax_6_3500_a=[0:0.1:Pmax_6_3500];
Pmax_6_0_a=[0:0.1:Pmax_6_0];
deltasoc_3500=1-(Pmax_6_3500_a/Pmax_6_3500);  % delta SOC at 3500 cycles
deltasoc_0=1-(Pmax_6_0_a/Pmax_6_0); % delta SOC at 0 cycles
figure()
plot(Pmax_6_3500_a,deltasoc_3500,Pmax_6_0_a,deltasoc_0)
grid on
title('Delta SOC as a function of Battery Power');
xlabel('Battery Power (kW)');
ylabel('Delta SOC');
legend('Cycle Number=3500','Cycle Number=0');


%% Part B Question-7
T_240=240; % K  Changing temperature changes design dependent variables
% Calculating new parameters with temperature change
io_240=121*exp(-4000/T_240);
D_240=7*exp(-6123/T_240);
Deff_240=(porosity^0.5)*D_240;
vsquare_240=((a*io_240*F*n)/(R*T_240))*((1/sigmaeff)+(1/Keff))*(lp^2);
v_240=sqrt(vsquare_240);
B1_240=((F^2*porosity*ci)/(R*T_240*(1-to)))*(1/(a*Cdl));
B2_240=((F^2*porosity*ci)/(R*T_240*(1-to)))*Deff_240*((1/Keff)+(1/sigmaeff));
B3_240=vsquare_240/B2_240;
Rct_240=(((2*(Keff/(Keff+sigmaeff)))*((tanh(v_240))/v_240))+(2*(sigmaeff/(Kef
f+sigmaeff))))*(sigmaeff/lp)^-1;
Rp_240=((2*(Keff/(Keff+sigmaeff)))+(2*(Keff/(Keff+sigmaeff)))*(((2*to)/((2*to
)+B2_240))+((B2_240/((2*to)+B2_240))*((tanh(sqrt((vsquare_240/B2_240)*((2*to)
+B2_240))))/(sqrt((vsquare_240/B2_240)*((2*to)+B2_240)))))))*(sigmaeff/lp)^-
1;
RcellA_240=Rohm+(Rct_240+Rp_240)*exp(x/y);
Rcell_240=RcellA_240/seperatorarea;
Rbatt_240=Rcell_240*M;
Pmax_7_3500=(((vmax*vmin)*(vmax-vmin))/(Rcell_240*(vmax+vmin)))*M/1000;  %
Max power at 3500 cycles
RcellA_7=Rohm+(Rct_240+Rp_240)*exp(x_6/y_6);
Rcell_7=RcellA_7/seperatorarea;
```

```
Rbatt_7=Rcell_7*M;
Pmax_7_0=(((vmax*vmin)*(vmax-vmin))/(Rcell_7*(vmax+vmin)))*M/1000; % Max
power at 0 cycles
Pmax_7_3500_a=[0:0.1:Pmax_7_3500];
Pmax_7_0_a=[0:0.1:Pmax_7_0];
deltasoc_3500_7=1-(Pmax_7_3500_a/Pmax_7_3500); % delta SOC at 3500 cycles
deltasoc_0_7=1-(Pmax_7_0_a/Pmax_6_0);% delta SOC at 0 cycles
figure(9)
plot(Pmax_7_3500_a,deltasoc_3500_7,Pmax_7_0_a,deltasoc_0_7)
grid on
title('Delta SOC as a function of Battery Power at T=240 K');
xlabel('Battery Power (kW)');
ylabel('Delta SOC');
legend('Cycle Number=3500','Cycle Number=0');

%% Part B Question-8

P_max_B8=25*10^3;
delta_soc_B8=((vmin-
vmax)+(((P_max_B8*RcellA3)/(M*seperatorarea))*(1/vmin+1/vmax)))/deltav; %
Delta SOC at 25 kW for 0 to 3500 cycles
batt_cap=MaxCap*abs(delta_soc_B8); % Usable Capacity of Battery
figure()
plot(cycle2,batt_cap)
grid on
title ('Usable Battery Capacity as a function of CN');
xlabel('CN');
ylabel('Battery Capacity (Ah)');

%% Part B Question-9

Rbatt_9=Rcell_CN_0_3500*M;
delta_9=((vmin-
vmax)+(((50000*RcellA2)/(M*seperatorarea))*(1/vmin+1/vmax)))/deltav;  % delta
SOC at pulse discharge power for 0 to 3500 cycles
C1rate_9=(OCVmax-vmin)*M./(Rcell_CN_0_3500*M-(deltav*M./((delta_9)*MaxCap)));
Eavailable_9=M*((MaxCap.*delta_9.*vmin)+(0.5.*MaxCap.*abs(deltav).*delta_9.^2
))/1000; % Available energy using C1 rate
figure()
plot (cycle2,Eavailable_9)
grid on
title ('Battery Energy as a funcition of CN');
xlabel ("CN");
ylabel ('Available Battery Energy (KWh)');
```