Analemmatic Sundial

Division By Zero

Designed by: Alex Watson, Hannah Gidianian, Caitlin Gundert, and Richard Thomas

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1. Problem Formulation

1.1. Introduction

The Problem Formulation phase features information regarding the general background of the Engineering Design project and the overall objective of the project simplified into a problem and a solution "black box" model (Figure 1-1).

1.2. Background

Division by Zero is a team of Environmental Resources Engineering students including Hannah Gidanian, Caitlin Gundert, Richard Thomas, and Alex Watson. The spring 2017, HSU Engineering Design Project conducted by Division by Zero features work done with client, Catherine Zane Middle School in Eureka, California in efforts to improve the school campus and implement a sundial in the front of the school. The purpose of the sundial is to educate children about the dynamic relationship between the earth and the sun.

1.3. Objective

The objective for this project is to design, create, and implement a fully functional sundial at Zane Middle School. The sundial will function in various ways, including telling the relative time of day with regard to the sun (the solar time) as well as the time of the year (season). The sundial will meet the criteria specified by the client representative at Zane Middle School, Mr. Kristopher Buihner.

Figure 1.1The Black Box Model developed by Division by Zero which addresses the client's problem and our solution to the problem.

2. Problem Analysis

2.1. Introduction

The Problem Analysis covers the specifications required by the project, as identified by the client. The specific criteria for sundial design are listed according to weight. Also, the details of the usage of the sundial, its implementation, and production volume are addressed.

2.2. Specifications

Specifications are requirements for the project that must be followed. There are three main specifications for the project which include: 1.) The location where the sundial is built 2.) The ease of interpretation by middle school aged children, and 3.) The ability of the sundial to tell time.

2.3. Criteria and Constraints

The criteria and constraints are standards by which the project can be judged. A weight value is assigned to each criterion by the client. *Table 1* features a list of the sundial's criteria and constraints.

Table 2.1 Various criteria for the sundial project and their relative importance by weight paired with the constraints of each.

Criteria	Weight	Constraint
Durability	10.0	The Sundial must withstand daily outdoor use.
Safety	9.0	The Sundial must be safe to use by children, unsupervised or supervised.
Educational Value	8.0	The Sundial must serve as a learning tool.
Multiple Time Components	7.5	The time of day and time of year must be measured by the dial.
Cost	6.0	The amount of money spend must not exceed $400 total.
Aesthetics	6.5	The Sundial must be visually appealing.
Interactivity	6.0	Children must be able to be physically involved while using the dial.
Accuracy	5.0	The sundial must display the correct solar time and season.
Sustainability	4.0	As many donated and/or recycled materials as possible must be used.
Precision	3.0	The hour must be within 16 minutes of the correct time, depending on the season.

2.4. Usage

The sundial will be used by teachers at Zane Middle School as a learning tool for students. Students and the general public can use the sundial at any time, but will be explicitly taught about how the dial works during class. The sundial will be primarily used when there is a sufficient amount of sun available for as long as the structure lasts.

2.5. Production Volume

One sundial produced for Zane Middle School is the total Production Volume for the project. The sundial is reproducible with the proper materials, and descriptive blue prints.

2.6. Literary Review

The literary review is the culmination of research and investigation about important topics that concern every aspect of a sundial. It is a summary of knowledge gained on solar time keeping devices.

2.6.1. Earth's Orientation to the Sun

The measure of the angle of the earth's axis relative to the sun is 23.4 degrees (Vincent, 2008) and is referred to as the obliquity of the ecliptic. The planets tilt is what causes the Earth to have seasons. For half of the year the northern hemisphere is bathed in light and the southern hemisphere is not. In the middle of either summer or winter the length of daylight is at its most extreme (AAD, 2014). Two days out of the year the sun will either shine for the most or least amount of time and that is called the solstice. During the summer solstice, the day is at its longest. Conversely, the winter solstice has the least amount of daylight in the year. Directly between these events the days and nights have values of time that equal each other. This happens twice a year and the occasion is referred to as the equinox (AAD, 2014). Figure 2.1 depicts the Earths seasons and how the tilted axis relates to the phenomena.

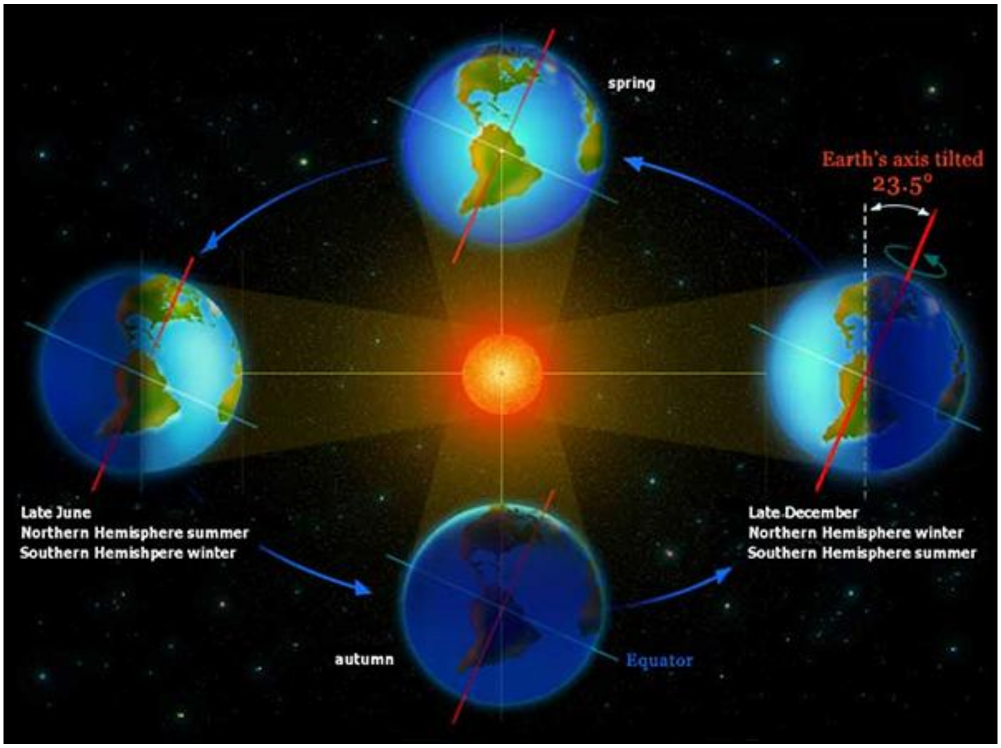

Figure 2.1 The distribution of sunlight on the earth over a year (Andrews 2013)

2.6.2. Ways of timekeeping

Humans created the idea of time as a unit of measurement between events. One way time is utilized is tracking the length of a day. A day is the periodic length between sunrises. But, because the Earth is round, people in different locations experience the sunrise at different intervals. The creation of time zones was a solution to account for the difference in longitudinal location (Waugh, 2012).

The imaginary line connecting the north and South Pole is called a meridian. At any moment in time the sun will be directly over a meridian. At this moment in time it is noon anywhere along that meridian. Experts refer to this way of timekeeping as Local Apparent Time. This is the type of timekeeping that a sundial utilizes. Two locations separated by 100 miles will have a different apparent time of 7.5 minutes (Vincent 2008). Local apparent time will result in shorter and longer days throughout the year because of the earth's tilted axis referred to in Figure 1. Another way of keeping time is called Local Mean Time, a variation that was created by taking the suns average speed across the sky. That average speed is equal to the velocity the sun would have in its elliptic. Four days out of the year the two kinds of time will be equal and have the same value. During all other instances the Local Apparent Time will read either slower or faster. The continental United States of has multiple time zones shown in figure 2.2, and are incrementally one hour in difference. Standard Time is another type of time. It is standardized from a single meridian and all other times relate to that location. Standard Time has no day to day inequalities of length because it is not localized. Standard Time separates time zones by 15 degrees where the entire zone uses the same time value. Some examples include Eastern Daylight Time, Eastern Standard Time, Greenwich Mean Time, and British Summer Time. A single moment in time can be expressed using these time zones four different ways (Waugh, 2012).

Figure 2.2 Time zones of North America (worldmaps.com 2016)

2.6.3. Types of Sundials

There are several different construction designs for sundials depending on the requirements and wants of the consumer. This section will discuss the design of different sundials, such as horizontal, vertical, and equatorial.

2.6.3.1. *Horizontal*

The flat horizontal sundial shown in figure 2.3 is commonly known as the "garden sundial." Parallel to the ground, it has a surface that receives the shadow on a horizontal plane. Most sundials have a gnomon, including the horizontal, which is a component varying in size that casts the shadow to show the time. The hour lines are on the outskirts of the plan and the shadow travels clockwise throughout the day (Bud, Sagwin, 2001).

An analemmatic sundial is an example of a horizontal sundial shown in figure 2.4 where a person is the gnomon. Two offset rows of time on this type of sundial are created to compensate for daylight savings. There are also different places where a person will stand depending on what month it is. This is because shadows are longer in the winter than they are in the summer ("Horizontal Sundial" n.d).

Using a horizontal sundial, the onlooker can read the time on the dial whenever the sun is shining (National University of Singapore, n.d). A disadvantage in building a horizontal sundial is in constructing the base. The base of the sundial is not perpendicular to the gnomon like it is on the equatorial sundial (which will be explained in a later section). The advantages to the sundials with the person as a gnomon are that they are more interactive. The disadvantage is that they can be a bit inaccurate at times. Since human shadows do not have a point and vary in size, it is hard to get an exact time.

Figure 2.3 Typical Horizontal Sundial (Alamy 2017)

Figure 2.4 Child's body replacing the gnomon in a horizontal sundial (“Sundials for learning” n.d.)

2.6.3.2. Vertical

A vertical sundial has the same concept as the horizontal in the way that there is a gnomon and the shadow radiates around the plane, but instead of the base being horizontal it is vertical. It is usually attached to a preexisting structure and is placed on the wall. A big difference the horizontal and vertical sundial is that a vertical sundial’s casted shadow rotates counterclockwise as the day goes on while the horizontal sundial shadow goes clockwise (Bud, Sagwin, 2001). The vertical sundial does not receive sunlight throughout the whole day. Notice the dial in Figure 2.5 is only able to tell the time during a certain section of the day (Bud, Sagwin, 2001).

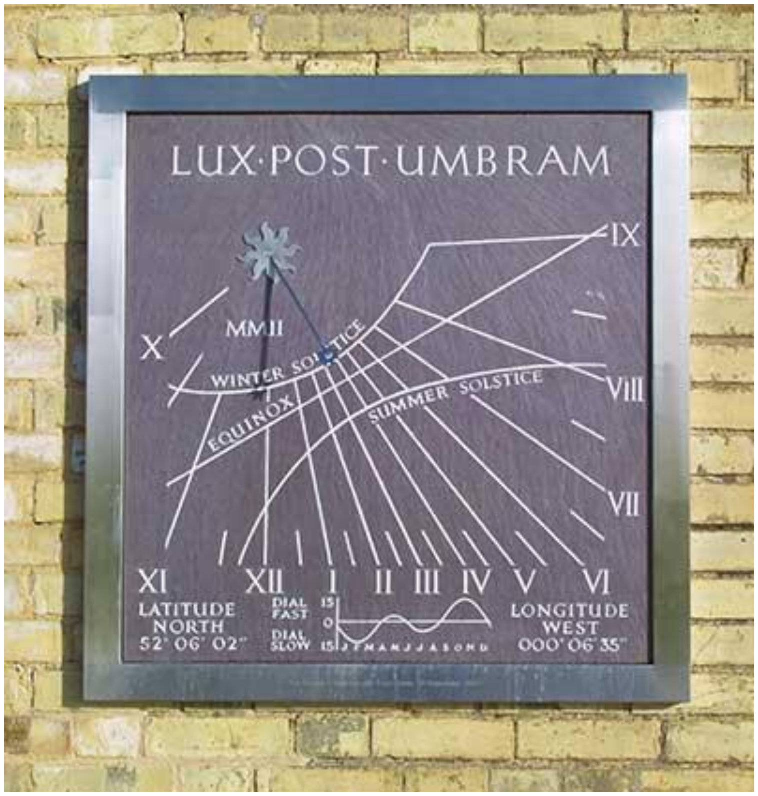

Figure 2.5 Vertical sundial planted on a brick wall (Harper 2017)

2.6.3.3. Equatorial

The equatorial sundial includes a leveled surface oriented with the celestial equator, and a perpendicular gnomon that points toward the North or South Pole. Examples include a strip in the shape of a circle cut in half seen in figure 2.7 or can be a full disk such as figure 2.6. When building the disk sundial the time is repeated on the bottom of the disk. As the season passes mid-summer where the shadow would be the shortest, it begins to lengthen when approaching the autumnal equinox. Once it passes the autumnal equinox and the sun is on the equator the shadow is then casted on the bottom of the disk. This is the reason for time to be displayed on both the bottom and top of the disk. The gnomon acts as the radial center of all time indicators. These indicators, often Roman Numerals, are located fifteen degrees apart from one another (Bud, Sagwin, 2001) and (Taylor, 1975). The incremental separations are calculated for using the 24 hours within each day for a 360-degree sphere that is the Earth.

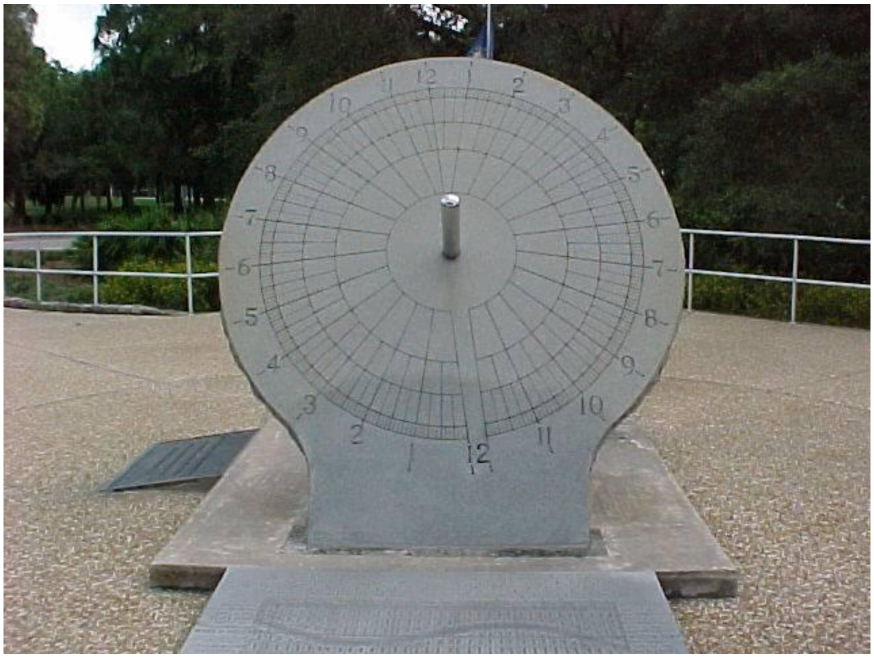

Figure 2.6 Disk shaped sundial ("Equatorial image gallery" n.d.)

Figure 2.7 Strip type equatorial sundial (Zorro 2011)

2.6.4. The Mathematics of Sundials

The sections below describe the mathematics involved in designing a sundial. Calculations on how time is calculated, specific gnomon heights and trigonometric functions are included.

2.6.4.1. Equation of Time

The Equation of Time is essential to understanding and relating the function of a sundial in Mean Solar Time to the Local Apparent Time, which is based solely on the shadow cast by the sun in a region without accounting for the time of year and the longitude of the region which is being analyzed by the sundial. Mean Solar Time is equal to the Local Apparent Time plus the Equation of Time. According to the figure 2.8, one must add or subtract the minutes on the y-axis for the date on the x-axis in order to convert to “clock time” (Sabanski).

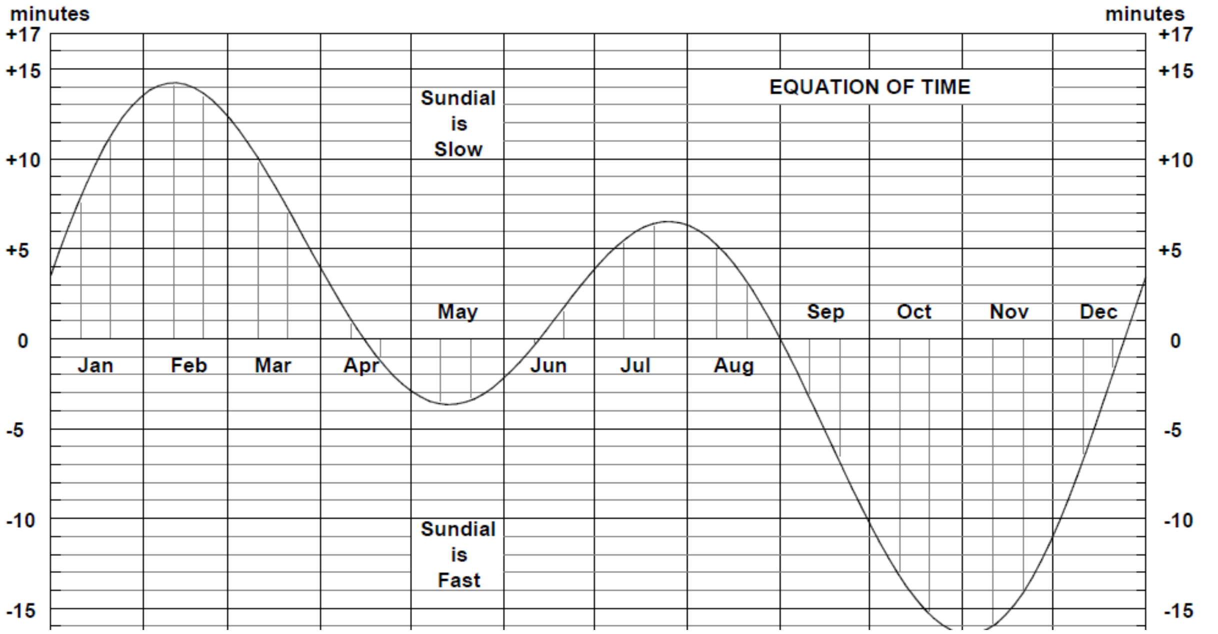

Figure 2.8 The Equation of Time and Longitude Correction (Sabanski)

2.6.4.2. Gnomon Angle and Height

The Gnomon is the instrument which is projected out of the face of a sundial in order to cast a shadow with the sun’s given angle. The shadow is at its shortest length when the sun is at its highest point, which is called solar noon. On the summer solstice, it will be shortest. You must multiply the minimum shadow length that is cast by the gnomon per unit of height, where units are choosable, by the degree of latitude which the sundial is positioned. The horizontal standard sundial will cast the shadow according to this calculation (Sabanski).

2.6.4.3. Trigonometric Functions of Sundials

Horizontal sundials, being one of the more common types, use the gnomon parallel to the axis of the Earth’s, which is tilted at approximately 23 degrees. There is a plane which is perpendicular to the gnomon which is projected onto the face or time telling display of the dial. Using a right triangle, one can calculate the relationship of the distance from the center of the dial face, perpendicular to the line which connects the plane to the northern pole of the dial face and the radius of the dial face, as well as the sine of the angle which is adjacent to the dial face radius, furthest from the center. (Vincent, 2008).

2.6.5. Pedagogy

Middle school students are the main users for the project. The sections below describe how a middle school student develops and along with how they learn.

2.6.5.1. Middle School Aged Development

Middle School is based upon the idea that children between the ages of 12 and 14 are at a critical state in their development and should be placed among others at the same stage in order to enhance the benefits of organized learning. One thing that students among this age have in common is the developing frontal lobe of the brain. This particular region is responsible for both making plans as well as judgment. Another key developing portion of the brain is that which is responsible for organizing, and critically analyzing complex thought. The average middle schooler uses their amygdala which is an emotional regulator (Caskey, 2003).

2.6.5.2. Interactive Learning

According to psychological research, adolescent children of the Middle School age learn best with classroom techniques that are compatible with their stage of brain development. This includes project based learning, group activities, and hands on learning. Being able to express oneself physically is critical to adolescent learning. Hands on activities involving physical movement or learning lab activities are some of the most effective tools for solidifying concepts in an adolescent (Caskey, 2003).

The process by which information is solidified in the brain of an adolescent child is the complex neural connection process. This process is strengthened by activity, instruction, and relating learning materials to real life. The natural curiosity that is encouraged by interactive learning is key to creating these neural connections that will impact the adolescent child permanently (Caskey, 2003).

2.6.5.3. Instructional Strategies

Certain strategies to address the learning style of students in Middle School are applied according to their developmental tendencies. There is an approach in child psychology that is called the Generation Effect. This is where a student is encouraged to generate answers to questions on their own after engaging instruction and retentional learning rather than the traditional method of reading a textbook and answering questions on a page. The statistics show a greater retention rate that is significantly higher. (Shore, 2013).

2.6.6. Materials

A variety of materials can be used to build a sundial. The sections below describe a variety of possible materials to be used building the sundial.

2.6.6.1. Steel

Steel is the combination of iron with carbon and other elements but mostly carbon. Steel is a very common martial used in construction because it has a high tensile strength (Bell, 2017). Tensile strength is the ability of a material to withstand weight. Steel undergoes the process of smelting. Smelting is the action of heating the metal to a high temperature and then quenching it in oil, the new material does not have any oxygen in it and is a harder material (Ohashi, 1992). Also, the addition of either nickel or manganese increases the tensile strength. There are many different types of steel including Carbon steel, Alloy steel and Standard steel. Where carbon steel is the most popular being made of mainly iron and carbon where alloy steel contains more chromium (Bell, 2017).

The durability of the steel is dependent on the other elements that are in combination to the iron and carbon. The addition of Chromium in the steel is the main element that combats the deterioration of the steel (Ohashi, 19992). If the location is in a coastal region the durability from salty humidity is important. The most durable of the steels would be stainless steel because there is an abundance of chromium in the material to form a layer of chromium oxide on the exterior of the metal. This layer protects the inner iron to prevent the formation of iron oxide, rust, and therefore extending the life expectancy of the metal (Paxton, 1997). Stainless steel is not impenetrable to deterioration but with proper care and installation the metal will last longer that other possible building materials.

2.6.6.2. Ceramic

Ceramic is a commonly used material in making paving stones because it has a very high melting point and is very strong. The most popular type of outdoor ceramic is porcelain (Barsoum, 1996). Due to the fact it is a very durable material against nature taking a very long time to decompose with the soil. Porcelain tiles are a clay based material that are made had by baking them at very high temperatures to make them stronger and able to deal with constant ware.

2.6.6.3. Concrete

Most construction concrete is a lime base material that is mixed with water to form a malleable material that can be molded and hardens over time. The main ingredients in concrete are mortars and plasters. Where the addition of water causes many reactions to occur at the same time where the products of those reactions will bond with each other to harden the material (Roy, n.d.). Also the addition of water is a very balanced mixture because the addition of too much water, although making it more malleable, makes it a weaker final product. And the addition of too little water doesn't allow the material to mix properly and the final result is very weak. The mixture of water has to be perfect to reach highest strength (Abercrombie, n.d.). When concrete is poured into a mold an addition of a solid like rebar is intended to increase the tensile strength against large loads.

2.6.7. Client's Needs

Zane Middle School asked for a large scale and interactive sundial. First the emphasized the importance for accuracy and asked for the sundial to be able to tell the time accurately, tell the date, and to tell when the solstice and equinox are. Another aspect that they originally wanted was a part of it to only be visible on one day of the year. Upon the second interview, they were interested in having the sundial on the ground marked in tiles so that the students stand on a mark and act as the gnomon or a safely constructed large scale gnomon that would be more accurate. But the biggest objective is that the sundial will last for a long time and be used for lessons.

3. Alternative Solutions

3.1. Introduction

Alternative solutions were developed during a brainstorm session and are discussed in this section. These feasible solutions meet the criteria of the project and fulfill the objective of making a durable sundial to learn from. Six solutions and an explanation of our brainstorm session are written below.

3.2. Brainstorm

One structured and collaborative brainstorming session was held at HSU inside of the Science D building. All team members were present and contributed to the process for the purpose of developing alternative solutions. Five minutes were allowed to conceptualize and conjure up new ideas about the existing styles of sundials. It was agreed that if an idea was shared a team member could only build on that idea rather than talk about its short comings. This process was recorded on a paper and can be referred to in Appendix A.

3.3. Alternative Solutions

The following is a detailed list of six alternative solutions for a sundial which were produced through two brainstorming sessions. The six alternative solutions are:

- Child Gnomon
- Pre-existing Structure as Gnomon
- The Open Book Dial
- The Garden Sundial
- Life Size Sun Clock
- Parabolic Sundial

3.3.1. Child Gnomon

The Child Gnomon design uses the shadow of the user to distinguish the time. The user stands on the corresponding month on the tile strip as demonstrated in part C in figure 3.1. The user then views their shadow to get the time of day. This design requires the student to interact with the sundial in order for them to tell time. The time and month tiles will be installed into a grassy area in the shape of a half circle as shown in part D in the figure 3.1. This solution resilient against harsh weather and will last the abuse from the students. There are two different sets of numbers representing the time based on it being the winter or summer months with A being the time for the winter and B for the time in the summer.

Figure 3.1 Sundial where the student casts the shadow.

3.3.2. Pre-existing Structure as Gnomon

Using the pre-existing structure as the gnomon allows for easy construction. This sundial design would be sustainable and work around the pre-existing environment. By using a structure like a basketball hoop as the gnomon makes a large-scale sundial that an entire class can gather around and observe a lesson increasing the education value. With the structure being larger, the chance that a child could move the gnomon is very small shown as part B in figure 3.2. The pre-existing gnomon has already passed the safety test of the school. The numbers would need to be measured and then painted onto the concrete as illustrated in part A. This alternative has the option of labeling the summer and winter times illustrated in part C so the sundial can be read correctly during daylight savings.

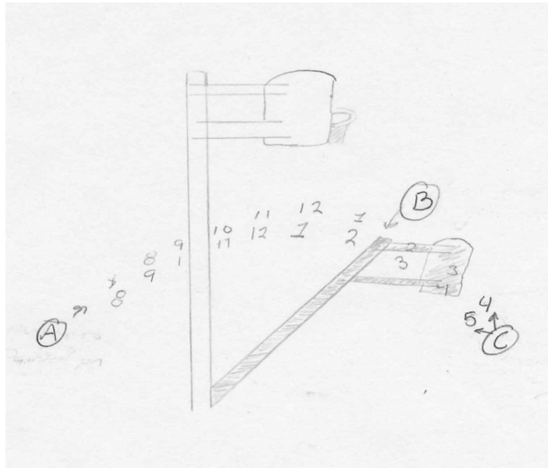

Figure 3.2 Sundial design where a preexisting structure casts a shadow

3.3.3. The Garden Sundial

The Garden Sundial is a horizontal sundial that tells the time of day. The gnomon and the base is made of metal and is raised on a concrete stand. The shadow of the tip of the gnomon, which protrudes at an acute angle as shown in figure 3.3, lands directly upon the corresponding time. This design will fit in virtually any place at Zane Middle School that is exposed to full sunlight. The convenience of this dial is that it is simple enough to be interpreted by the children and materials used to build the dial can come from sustainable sources.

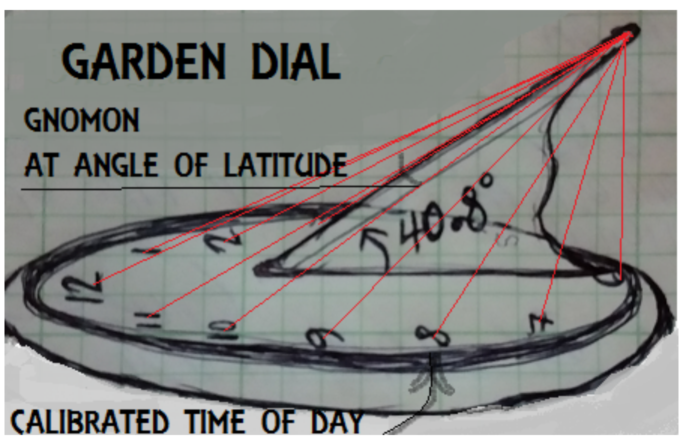

Figure 3.3 The typical sundial found in gardens. The base is horizontal and gnomon protrudes at an angle.

3.3.4. Open Book Dial

The Open Book Dial is designed in a shape of a book. This particular dial features a gnomon that stands perpendicular to the face of the book, but runs parallel along the inner spine. This design gets rid of the protruding gnomon typically found on sundials reducing the safety risks. This sundial casts a basic shadow either left or right of the center of the opened book, depending on the time of day as shown in figure 3.4. When the sun is at its highest point, it shows solar noon, which is directly below the gnomon. The book is slanted at the exact latitude of the location in order to receive an accurate reading. The sundial is designed to tell the user what month they are in. The gnomon contains a hole in the center of the gnomon and based off the position of the sun the holes shadow falls on a designated arc. This aspect gives the user another time component.

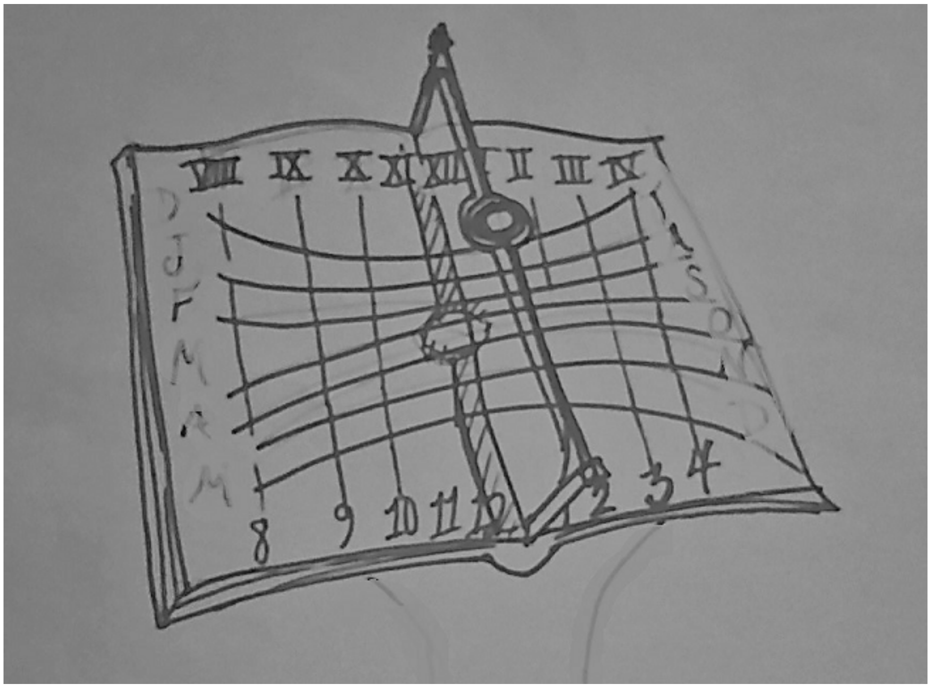

Figure 3.4 Open Book Sundial where the gnomon is connected from top to bottom and casts a shadow that can tell month and time.

3.3.5. Life Size Sun Clock

Life Size Sun Clock is a large scale horizontal sundial. It is in a shape of a clock with the rim used as a pathway. Benches can be added to make the area a hangout locations that is educational and aesthetically pleasing for students. The gnomon is a large pole made of a durable metal and casts a shadow on the stone path way. Where the gnomon is held up, shown in figure 3.5, there is a plaque explaining how the time and month is read. This aspect gives students a chance to understand how to use the sundial without a teacher present. The time tiles are level with the ground and are made with tiles reducing the safety risks. Grass is put on the inside of the clock to increase aesthetic value.

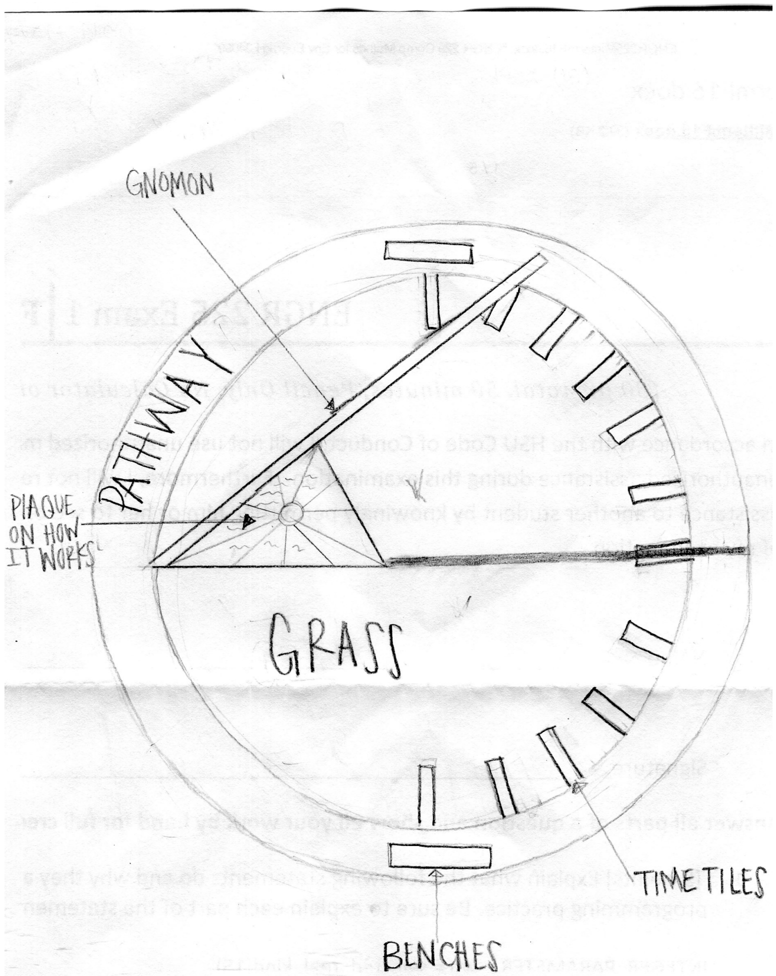

Figure 3.5 Life Size Sun Clock where the gnomon is a large structure and the dial is in the shape of a clock.

3.3.6. Parabolic Sundial

This Parabolic Sundial structure is constructed out of steel. A steel framework makes this design very resilient to weather and destructive students. The gnomon is connected to the frame on both ends as shown in figure 3.6. This aspect makes the structure safe for students to be around as there are no sharp points or edges. The face is curved in such a way that the gnomon's shadow lies directly over the designated time of day. Two rows of time intervals are printed on the face and offset. The second row is designed to read in the other half of the year when the sun is in a lower position in the sky. The offset is created for a very similar idea. This aspect allows the sundial to be read correctly during day light savings fulfilling the multiple time component criteria.

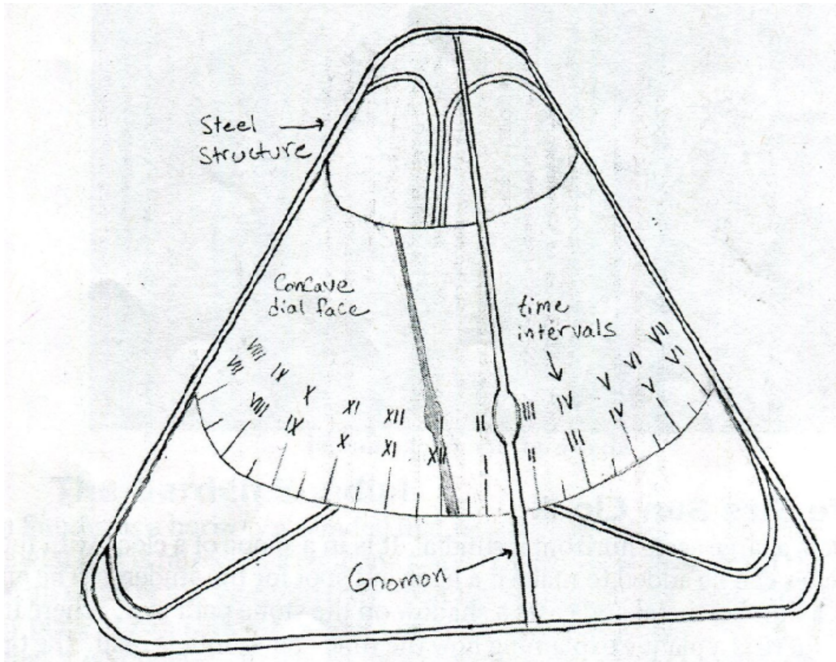

Figure 3.6 Parabolic sundial with connected gnomon

4. Decision Phase

1. Introduction to the Decision Phase

All alternative solutions from section 3 are considered in section 4, the decision phase. The Delphi multiplier, which considers all the criteria discussed in section 2, is used create the final design.

2. Criteria Defined

Definitions of the criteria are presented below. These definitions are used to judge how well each alternative solution fits the criteria.

Cost- The cost of all the materials needed to build the project will not exceed the predetermined budget mentioned in section 2.

Durability- The structure will be able to withstand all weather conditions and abuse from students.

Aesthetics- The project needs to look aesthetically pleasing to visitors and students who come to the school.

Sustainability- Materials used need to come from ecofriendly sources and not contribute to environmental issues.

Interactivity- The structure needs to be able to get the children engaged while learning a new topic.

Safety- Students will constantly be around the structure and the project must be free from safety hazards discussed in section 2.

Accuracy/Precision- The project must be an accurate presentation of solar time. The structure is able to give a precise casted shadow to tell the time.

Multiple Time Components- The structure has components that will tell time, month and seasons.

Educational Value- The project will be used as a teaching tool by teachers and will be able to instruct students about the position of the earth with respect with the sun.

3. Solutions

The following alternative solutions, outlined in Section 3, are considered in the Delphi Matrix

- Child Gnomon
- Pre-existing Structure
- Open Book Dial
- Garden Sundial
- Life Size Sun Clock
- Parabolic Sundial

4. Decision Process

The Delphi matrix was used as a decisive tool to determine the final solution. Each individual team member completed a Delphi matrix. First the criteria, mentioned in section 3, were rated on a scale from 0-10, on a level of importance with 10 being the most important. This is shown in table 4.1. Each solution was then rated on a specific aspect that the criteria were addressing, on a scale of 0-100, and then multiplied the number by the importance level shown in the table 2 by a diagonal line through that specific column. The sum of all scores in the chart is in the final row and was used to determine the best solution. The average of the criteria ratings along with the solution ratings was done, creating the final matrix found in table 4.2.

Table 4.1 Criteria mentioned in section 2 weighted on a scale of 1-10 based off of importance.

Criertia	
List	Weight
Durability	10
Safety	9
Educational Value	8
Polychronicity	7.5
Cost	6.5
Accuracy	6
Interactivity	6
Aesthetics	5
Sustainability	4
Precision	3

Table 4.2 Delphi Matrix used to in the decision process

Criteria	Weight	Alternative Solutions (0-100 high)				
		Child Gnomon	Pre-existing Structure	Open Book Dial	Garden Sundial	Life Size Sun C
Accuracy	6	62.5 / 375	55 / 330	90 / 540	77.5 / 465	67.5
Aesthetics	5	75 / 375	55 / 275	85 / 425	65 / 325	77.5
Cost	6.5	82.5 / 536.25	86.25 / 560.625	52.5 / 341.25	58.75 / 381.875	51.25 / 33
Durability	10	92.5 / 925	67.5 / 675	72.5 / 725	67.5 / 675	77.5
Educational Value	8	78.75 / 630	57.5 / 460	72.5 / 580	47.5 / 380	62.5
Interactivity	6	100 / 600	32.5 / 195	43.75 / 262.5	35 / 210	55
Polychronicity	7.5	53.75 / 403.125	30 / 225	70 / 525	50 / 375	65
Precision	3	56.25 / 168.75	45 / 135	82.5 / 247.5	70 / 210	63.75 / 1
Safety	9	97.5 / 877.5	85 / 765	73.75 / 663.75	63.75 / 573.75	68.75 / 6
Sustainability	4	80 / 320	83.75 / 335	60 / 240	55 / 220	57.5
	Total	5211	3956	4550	3816	4258

5. Final Decision

The child gnomon solution is the highest scoring design in the matrix shown above and is therefore the best fit solution. This design is a durable interactive teaching tool that will be safe for the students to use. Multiple components will be added to tell both time and what the season it is.

5. Specifications

5.1. Introduction

Section 5 includes a description of the final solution chosen from the previous section including detailed pictures to give an in-depth representation of the solution. The layout and arrangement of the components are described in depth. A detailed table of cost is included to analyze the hours spent in designing the solution, the construction and the maintenance that will be implemented. Section 5.4 includes instructions on how to use the final solution.

5.2. Description of Solution

An analemmatic Sundial is a horizontal sundial that does not use a stationary gnomon. Instead there is a labeled area for the user to stand on based on the month. In the allotted 10’ by 10’ area, the dimensions for the sundial fit into a 9’ by 5’ rectangle because the orientation of the square given to us does not align with north and south. The hour markers are on an elliptical pattern shown in figure 5.1.

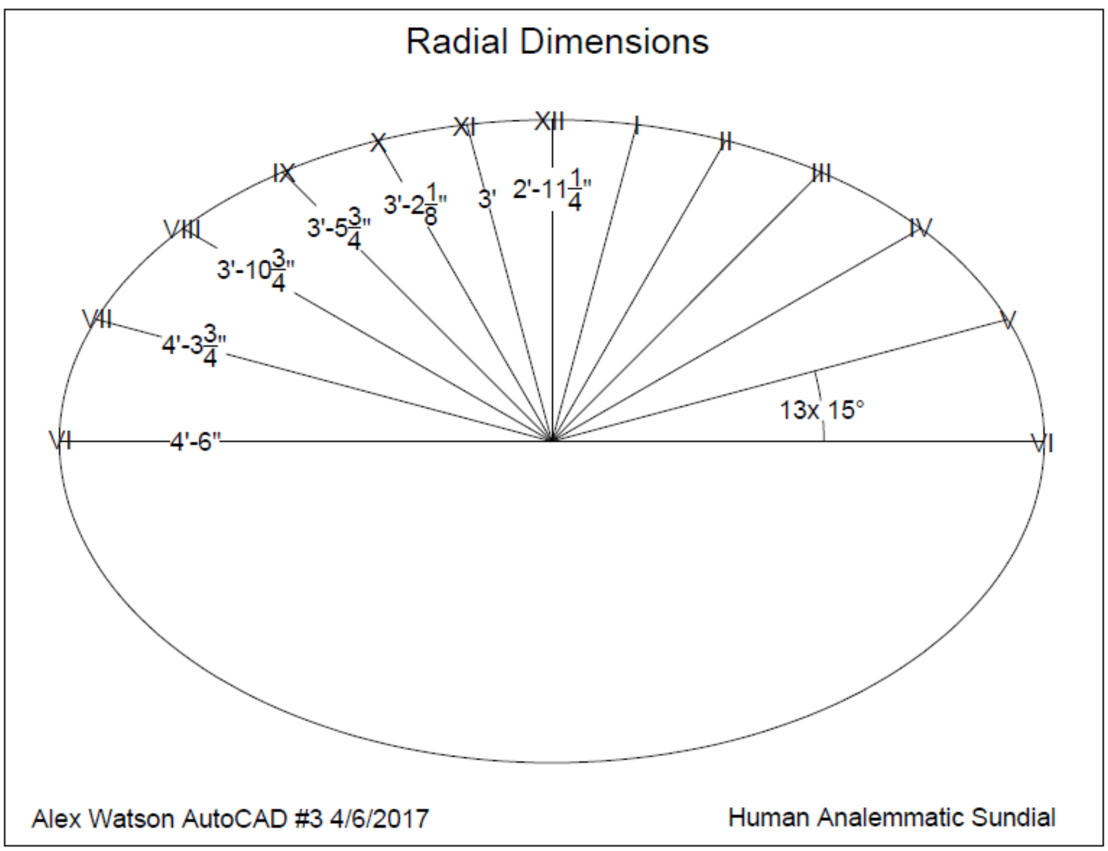

Figure 5.1 Sundial Dimensions (Watson)

5.2.1. Alignment of North and 12

An important aspect of this sundial is that the vertical axis needs to be aligned with true north and not magnetic north. The 12-hour marks true north with 6 AM pointing due west and 6 PM being due east. The formula to find the angle of each respective hour mark is as follows:

$\tan \Theta = (\tan(15\mathring{x}\ t))/\sin(\Phi)$

The variable, "t" is the time in hours, Φ is the geographic latitude and Θ being the angle of the given line. This equation states each hour mark or Roman numeral will be 15 degrees away from one another.

5.2.2. Month lines cast by a Removable Gnomon

The month lines are given by the following formula:

$\Theta = 90^{\circ} - (\text{Latitude} - \Delta)$ where $\Delta = -23.45^{\circ}(\cos((360/365)x(d+10)))$

The variable, "d" is the number of days since January 1st. These calculations were performed for the first of every month based on the shadow cast by a 36 in. post. The concept being that the shadow being cast fall between two lines showing what month it is.

5.2.3. Indentations in the concrete

Indentations of all the hour lines are made into the concrete while it is still wet. There are imprints for all numbers in Roman numerals. The impressions will be 0.25 inches deep. The indentations for the months will be 0.125 inches deep so that there is more definition in each letter.

5.2.4. Building Roman Numeral Stamps

An outlined stencil cut out of card stock is necessary prior to tracing onto a piece of plywood. The outline of each Roman numeral is then cut using a jig saw. Each Roman numeral is then sanded to a smooth surface and the edges are rounded, as shown in figure 5.3.

A backboard for each number is cut by laying down the roman numerals on the backboard to gauge size of backboard needed. Traced using pencil are each of the numerals onto this backboard, each set of characters has to be flipped backwards to be read correctly when imprinted into the concrete figure 5.2 for the number eight. After tracing, pilot holes are drilled for each character of the number through the backboard. Screws are drilled through the backboard into the numerals to secure them to the board.

Figure 5.2 Fully completed sundial time stamp

Figure 5.3 Plywood Roman numeral cutouts

5.2.5. Building Standing Position Stamp

Using 2 x 1 pieces of fir trim, the outline is cut, and centered at the horizontal axis of the standing position grid. Each corner point is fastened together to provide stability to the corners by screwing in a triangular piece of ¼ inch plywood. The distance from the center axis is measured to the outer wall and pieces of wood are fitted to the area. The distance is measured from the base of the grid for the displacement of each line. These pieces are screwed in from the outside to secure in frame. The final stamp is shown in figure 5.4.

Figure 5.4 Finished standing position stamp

5.2.6. Building stamps for Month

Using the wooden letters, the area needed for each month on a piece of plywood can be gaged to determine the size of board needed. The letters are thickened, two-ply to double the thickness and create a more effective, deeper indentation. This thickness is shown in figure 5.5 These letters glued to the backboard are mirrored so that they are read correctly when imprinted into the concrete, as shown in figure 5.6.

Figure 5.5 Laminated wood lettering

Figure 5.6 Fully completed month letter stamp

5.2.7. Impressing into Concrete

While impressing the wooden stamps into concrete, a releasing agent must be used in order to ensure that the concrete does not stick to the stamp. Canola oil works as a releasing agent. The wooden stamps soaked in canola oil will prevent the impressions from defecting the concrete.

5.3. Costs of project

5.3.1. Design Cost Hours

The design cost hours indicate the total number of hours that each member of the Division by Zero team spent on this design project. This includes client meetings, project design, and building. The relative amount of time spent on each aspect of the project are representing in figure 5.7.

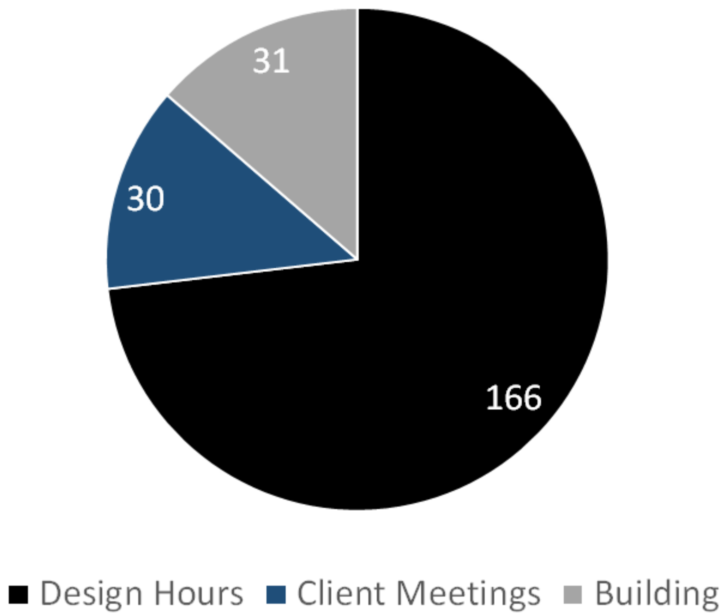

Figure 5.7 Distribution of project time

5.3.2. Design Expenses

The total cost of materials used in the construction of the analemmatic sundial was $112.44. Represented in Table 1, we were able to stay within our budget of $400.00 because more expensive materials were donated and we were not required to buy the concrete.

Materials	Quantity	Cost Per Quantity $	Our Cost $
Table Cloth	1	17.99	17.99
Canvas	1	29.99	29.99
Dowels	2	0.69	1.38
Wood Letters	6	3.99	23.94
1x2 Furr Trim	1	30.99	30.99
1/4 in Plywood	1	Donated	Donated
Wood Glue	1	4.99	4.99
Sand Paper	4	0.79	3.16
		Total Costs	112.44

Table 5.1 Total cost of building materials

5.4. Instructions for implementation

Construction begins with the angles of the numbers marked out with chalk on the surrounding dry concrete based on the origin of the sundial. From the origin, a small sleeve for inserting the removable pole is placed into the dirt as a reference point for when the concrete is poured. Once the concrete is poured, the impressions are made with the stamps and the foot placement grid is imprinted into the concrete from a large stamp that we constructed. The month lines are then troweled into the concrete from measurements made, while the construction worker is sitting on the grid stamp to impress it into the concrete. Once constructed, the concrete will need to be left to dry.

5.5. Results

The results of building the designed model is that the sundial is an effective way to teach children about solar time. The final design has not protruding objects that could harm any user and is a low cost solution to building a sundial. By using the concrete, the sundial is very durable for users of all ages. The removable gnomon allows the teacher to demonstrate the suns path across the sky throughout the year. See appendix C for a comprehensive review of the final results.

6. Appendices

6.1. Appendix A

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6.2. Appendix B

6.3. Appendix C