

# River Outlet with Hydroelectric Power Generation for the Auburn Dam

*Prepared for*

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## **Abstract**

The objective of this project was to design an operational river outlet system that could sustain maximum flows of 5200 cfs and generate 4 MW of power. To meet the objective, an analysis was done to determine the theoretical power output at the maximum flow, the diameter of the penstock, and the height and diameter of the surge tank to recommend a design for the system. This system incorporated 5 different components which were: 1) Penstock, 2) Head gate, 3) Surge Tank, 4) Turbine, and 5) Tailrace. The recommended turbine for the system was a Francis turbine with theoretical power capabilities of 144 MW at maximum flow. The analysis also determined that the design parameters should include a factor of safety to account for natural disasters and avoid catastrophic failure.

# Table of Contents

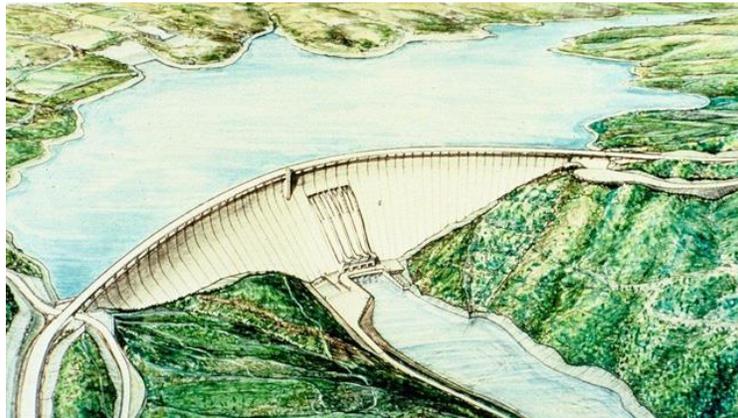
Abstract.....	ii
Table of Contents.....	iii
Table of Figures.....	iv
Introduction.....	1
Overview of the Problem.....	1
Objective.....	2
Importance of Solving the Problem.....	2
Approach to Solving the Problem.....	2
Design Steps .....	3
Problem Identification .....	3
Design Statement.....	3
The Model.....	6
Application .....	6
Parameters.....	7
Sensitivity Analysis .....	7
Results.....	8
Discussion.....	11
Conclusions.....	12
References.....	14
Appendices .....	15
Appendix 1: Technical Methods and Application .....	15
Appendix 2: Raw Code.....	17
Appendix 3: Raw Data.....	22
Appendix 4: Design Brief.....	25
Appendix 5: Copy of Technical Memo to Client .....	27
Appendix 6: Annotated Bibliography including all of your references.....	31

## Table of Figures

Figure 1: Artists rendition of the Auburn Dam courtesy of the Army Corps of Engineers (KQED 2014)...	1
Figure 2: General diagram of the river outlet system with the turbine located at the bottom of the length of the intake pipe.....	2
Figure 3: Flow chart of the general design process .....	3
Figure 4: The Bartlett dam with an overflow spillway headgate currently in the open position (Stephens 2004).....	4
Figure 5: General diagram for the operation of a turbine for electrical power generation (USGS, 2019)....	5
Figure 6: A general design of the system with all discussed components. Modified from (Ramazen et al. 2012).....	6
Figure 7: Penstock diameter (ft) vs Flow Rate (cfs).....	8
Figure 8: Theoretical power output (MW) and Reynolds number vs flow rate (cfs) for the system given initial conditions .....	9
Figure 9: Sensitivity analysis results of percent change of three parameters effect on penstock pipe diameter. ....	10
Figure 10: Design plan for the system based on the parameters in Tables 1 and 3.....	11

## Introduction

This analysis was done to determine an operational system to meet the needs of the United States Bureau of Reclamation (USBR) in facility power generation of the Auburn Dam. The Auburn Dam was originally proposed by the USBR in 1964 and began construction in 1968. Construction was halted in 1978 by a 5.7 magnitude earthquake that knocked down the each based cofferdam. Even with adjustments to the plan being made to account for seismic activity, interest was lost and water rights were revoked in 2008. The client now wants to create a new proposal for the Auburn Dam including a river outlet system for facility power generation. Figure 1 below shows a sketch of the dam from the army corps of engineers.

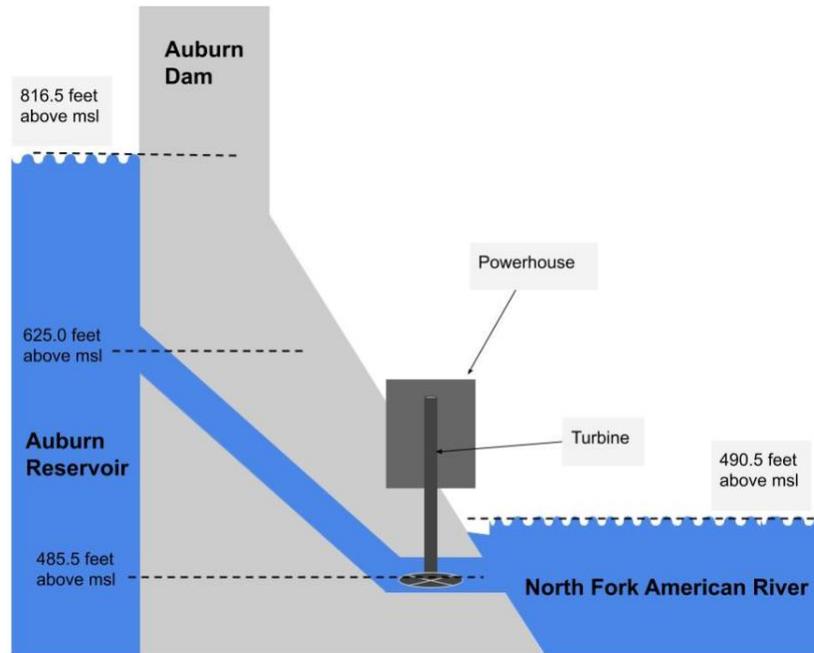


**Figure 1: Artists rendition of the Auburn Dam courtesy of the Army Corps of Engineers (KQED 2014)**

## Overview of the Problem

Hydraulic turbines are an effective way to generate power using an environmentally sustainable approach. Dams as a whole are a way to not only store water but also create power that is then transferred to the grid. In this situation, the 800mw of power that would be generated by the Dam's main hydroelectric system would go directly to the grid and not be utilized for facility operations. River outlets which is used to maintain a constant flow downstream of the dam, even in times of low water surface elevation. By placing a 4mw turbine in the river outlet, this operational power requirement can be met sustainably.

The problem solved in this analysis was to determine the basic design parameters of a river outlet system that utilized a turbine for power generation. This power would then be used to power all the operations of the unit. The basic design parameters were identified as pipe material, turbine type, and required pipe diameter. A general diagram of the system provided by the client is available below in Figure 2.



**Figure 2: General diagram of the river outlet system with the turbine located at the bottom of the length of the intake pipe**

### Objective

The goal of this project was to provide information to the client regarding the design of the river outlet system based on their specified criteria. The five main considerations of the design were determined as: 1) Penstock, 2) Head gate, 3) Surge Tank, 4) Turbine, and 5) Tailrace. The objective of this project was to design an operational river outlet system that could sustain flows of 5200 cfs and generate 4 MW of power.

### Importance of Solving the Problem

The importance of solving the problem is to provide the client information for how to construct an operational river outlet system. The system designed needed to generate the 4 MW of power required for dam operations. The design of this system also needed to ensure that the downstream portion of the north fork of the American river would have a sufficient flow rate, even in times of low water surface level.

### Approach to Solving the Problem

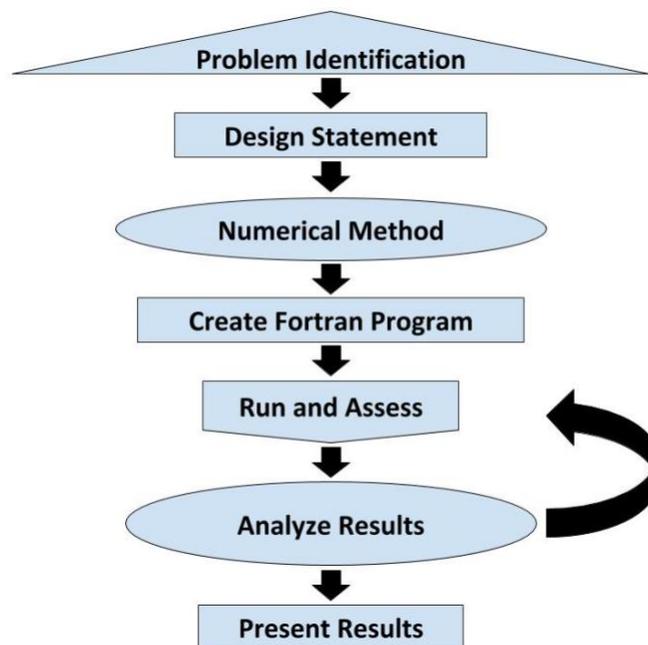
The Dam in question will be constructed at mile 20 of the north fork of the American river between the towns of Auburn and Cool. The river outlet system would have an inlet elevation from the reservoir of 625 feet above msl (Mean Sea Level) and discharge in the downstream portion of the river at 485.5 feet above msl. The main parts of the design that needed to be considered to create an operational system were defined as: 1) Penstock, 2) Head gate, 3) Surge Tank, 4) Turbine, and 5) Tailrace.

The turbine used for power generation will also be researched based on different systems available, considering client specifications to provide a recommendation. Once these design elements are chosen, a fluids based analysis can be done employing the secant method to determine the pipe diameter.

This report includes 4 sections: 1) Design steps, 2) Application, 3) Results, 4) Discussion, and 5) Conclusion.

## Design Steps

This section goes through the process by which the system was designed given the client's needs and provided constraints. Below, Figure 3 shows the general process taken to design the system.



**Figure 3: Flow chart of the general design process**

### Problem Identification

The problem was that the original design plans of the Auburn Dam did not include a system for power generation for facility operation. A modification of the river outlet system designed for discharge into the river at reservoir water surface levels below inactive storage, 816.5 feet, provided a solution that met the facility power demands.

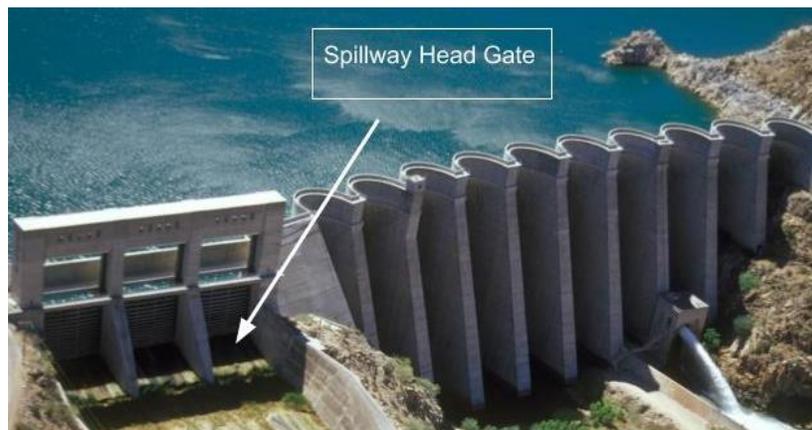
### Design Statement

The design of the system had to meet the facility power requirement of 4MW as well as a maximum flow rate through the system of 5200 cfs. The design of this new system would have four main components: 1)

Penstock, 2) Head gate, 3) Surge Tank, 4) Turbine, and 5) Tailrace. The use of these components allowed for the system to meet all client specifications.

1) Penstock: The penstock is the delivery pipe system by which water is sent toward the turbine. These pipes are usually relatively thick and made of a strong material such as steel or iron. The pipe should have no sharp bends or turns if possible to avoid water hammering or head loss (Linsley and Franzini 1964).

2) Head Gate: A head gate is used to stop the flow of water from the penstock before the turbine to allow for repairs on different components on the system. This component can also be utilized to decrease the flow of water entering the turbine to reduce power generation and cause less turbulent flow. Turbulent flow is determined by the Reynolds number value being greater than 4000 (White, 2008). Turbulent flow is generally harder on turbines due to the potential for sediment to enter the system and erode inner components.



**Figure 4: The Bartlett dam with an overflow spillway headgate currently in the open position (Stephens 2004)**

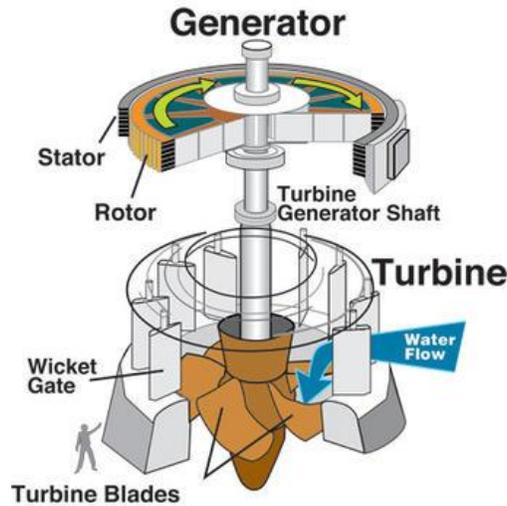
3) Surge Tank: The surge tank is a pipe that is used to transfer energy once a valve or gate is closed. If this energy is not dissipated it can cause a water hammering effect on the pipe and gate leading to system damage and potential failure. A water hammering effect occurs when flow rate is quickly decreased (McStraw, 1996). Surge tanks both absorbed this pressure energy as well as provide water to the system once operation restarts.

4) Turbine: A hydroelectric turbine is a device that converts the kinetic energy of a fluid to mechanical energy that can be used to power facilities. These turbines have a system by which the water rotates a wheel like system to transfer energy. Turbines all have different operating head ranges as well as maximum power and efficiency's. Based on the constraints provided by the client, three different turbines were considered initially: 1) Pelton wheel, 2) Francis Turbine, and 3) Kaplan Turbine. These three options

provided a range of values for operational parameters so the correct turbine could be chosen. The operational parameters for the different types of turbines considered in the analysis are available in Table 1 (Dritna and Sallaberger 1999). A general diagram representing the operation of a turbine is available below in Figure 5.

**Table 1: Operational parameters for the three turbine types considered in the analysis (Dritna and Sallaberger 1999)**

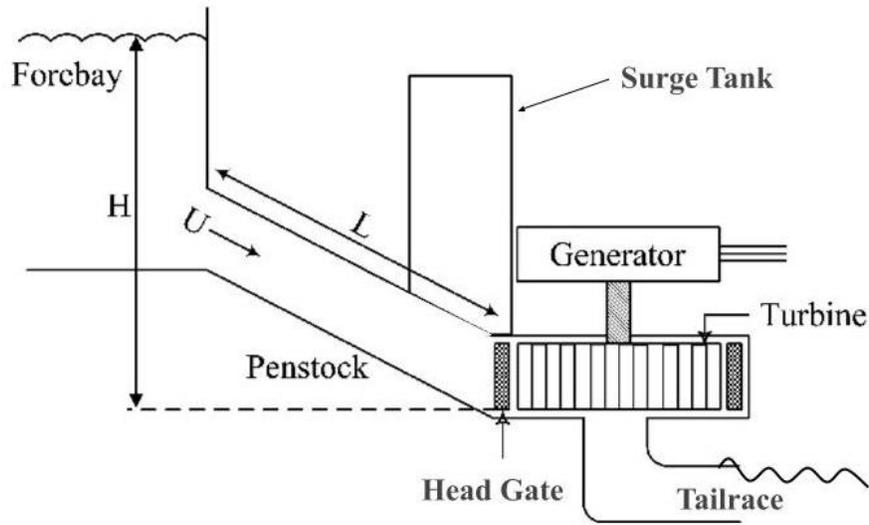
<i>Turbine Type</i>	<i>Pelton Wheel</i>	<i>Francis Turbine</i>	<i>Kaplan Turbine</i>
<b>Operating Head (ft)</b>	300-5100	240-1500	1-1200
<b>Maximum Power (MW)</b>	300	1000	200
<b>Highest Efficiency (%)</b>	93	94	94
<b>Operating Flow (ft<sup>3</sup>/s)</b>	1.5-150	2.1-1850	4.5-3000



**Figure 5: General diagram for the operation of a turbine for electrical power generation (USGS, 2019)**

5) Tailrace: The tailrace is the system by which the water that passed through the turbine is discharged into the downstream end of the river (Linsley and Franzini 1964). In situations that the turbine and powerhouse system are close to the river, outflow can be released directly into the waterway. If the system is further away however, a channel or pipe system must be employed to make sure the water can be discharged into the stream. Tailrace design is a way to help conserve gross head of the system.

These components all served a different purpose at various stages of the system to allow for power generation and river discharge in water surface levels below inactive storage. Figure 6 below shows a diagram of the system with all discussed components.



**Figure 6: A general design of the system with all discussed components. Modified from (Ramazen et al. 2012)**

### The Model

The model used to determine the minimum pipe diameter required at the maximum flow rate was a Fortran program that utilized the secant method. The secant method is a root finding method that is implemented when dealing with more computationally difficult functions. More information on the numerical method is available in Appendix 1.

A numerical method was employed due to the complexity of the function formulated to solve the problem. Some mathematical equations are too computationally difficult to solve by hand so a computer algorithm utilizing a chosen numerical method is employed to achieve an accurate estimate. For this problem, a Fortran program was created to employ the secant method.

In order to create the Fortran program, first all parameters had to be determined. Then the equations and their relationships had to be defined so only one unknown remained. These defined equations were then placed in a function that was solved by a secant method subroutine. Raw code for the program is available in Appendix 2. The model provided results as a single value that represented the diameter of the penstock in feet.

### Application

The problem had two parameters that were provided by the client that provided the guide for all design and problem formulation. These Parameters were flow rate ( $Q$ ) and Facility Power Demand ( $P$ ). The only

values missing in the problem formulation was stopping criteria, which was determined at the discretion of the programmer to maintain computational efficiency.

### Parameters

A few sources of variability in the model would be parameterized to isolate the problem in question (river outlet pipe diameter). Some of these parameters were established based on fixed positions of components in the system, such as the reservoir, the discharge elevation into the river, and the length of the penstock (L). Others were determined by physical properties of the material such as the Friction Factor (f) and Roughness of the pipe material ( $\epsilon$ ). Values for all parameters are available below in Table 2.

**Table 2: All parameter values for the problem formulation and model**

Parameter	Description	Value (if set)	Units
Hr	Water surface elevation in reservoir	816.5	Feet
Hi	Intake Elevation	625.0	Feet
Hd	Discharge Elevation	485.5	Feet
He	River Elevation	490.5	Feet
f	Friction Factor		Unit less
L	Length of the penstock	580	Feet
D	Diameter of penstock		Feet
Ds	Diameter of the surge tank	4*D	Feet
Hs	Height of the surge tank		Feet
Q	Maximum Flow Rate	5200	ft <sup>3</sup> /s
V	Fluid Velocity	Q/Area	ft/s
Re	Reynolds Number		Unit less
P	Facility Power Demand	4	MW
$\epsilon$	Roughness of Steel	0.00015	Feet

### Sensitivity Analysis

A sensitivity analysis was performed on the three key parameters of the analysis: 1) Q or the flow rate of the water from the reservoir, 2) L or length of the pipe, and 3) Hi or system intake elevation. The sensitivity of these parameters was determined by changing the magnitude of the parameter by percentages of -20% to 20% by increments of 5%. This value was then used to calculate a new pipe diameter in the model that was compared to the actual value and reported as a percent change. The equation used to determine percent change is shown below.

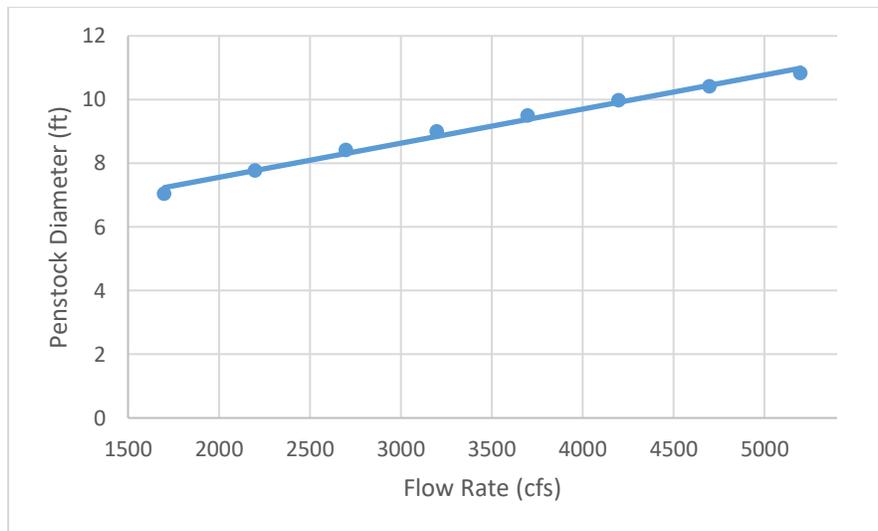
$$\text{Percent Change} = \frac{(\text{Actual Value} - \text{Altered Value})}{\text{Actual Value}} * 100 \quad (1)$$

A sensitivity analysis was also done on power generation (P) relative to changes in the flow rate of water through the system. The clients estimated energy needs for facility management was 4MW. An estimation of theoretical power generation was calculated from a minimum flow of 200 cfs to 5200 cfs at increments of 500. A graph of this relationship was then created to allow the client to formulate a regulatory flow management schedule.

## Results

Results were determined for all parameters necessary to design a functional system for the client’s facility energy requirement for operation. The model created in Fortran was employed to determine the minimum diameter of the penstock system. This value was then used to determine the specifications of the surge tank as well as the type of turbine to create at least the minimum energy requirement of the dam. Diameter and height were reported in feet and power was reported in megawatts.

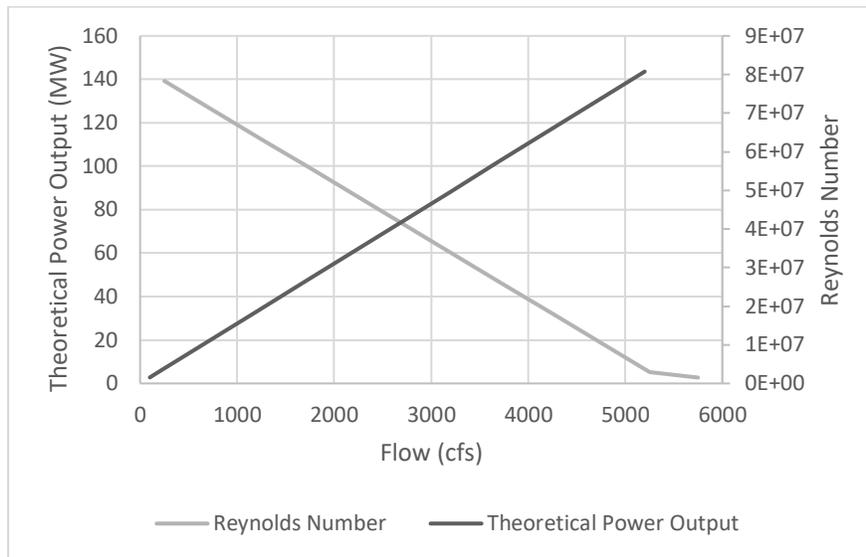
Penstock diameter was calculated using the Bernoulli equation in conjunction with the Colebrook equation to iterate to a reasonable estimate. Information on these equations is available in Appendix 1. The minimum pipe diameter of the penstock to allow for power generation of a minimum of 4 MW at the maximum flow of 5200 cfs was 10.83 feet. The relationship between pipe diameter and flow for values between 1700 and 5200 cfs is available below in Figure 7.



**Figure 7: Penstock diameter (ft) vs Flow Rate (cfs)**

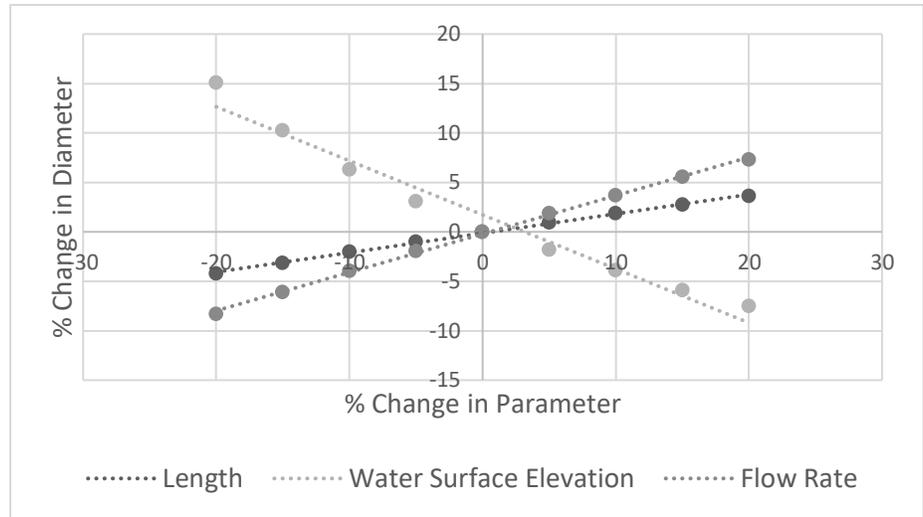
In order to determine the optimal turbine for the system given the parameters and constraints from the client, the theoretical power output was calculated for a range of flows from 200 to 5200 cfs. The elevation head from the water in the reservoir was determined to be 326 feet. This head value as well as a maximum flow rate of 5200 cfs made the Francis turbine the obvious choice for the system. Theoretical

power output and Reynolds number vs flow rate is available below in Figure 8. Equations for these calculations and descriptions are available in Appendix 1.



**Figure 8: Theoretical power output (MW) and Reynolds number vs flow rate (cfs) for the system given initial conditions**

Sensitivity Analysis Results: The sensitivity analysis was done on three chosen parameters of the system: 1) Length of the penstock, 2) water surface elevation height, and 3) Flow rate. Each parameter was changed from arrange from -20% to 20% by increments of 5%. Those values were then ran back through the model to observe the effect on the penstock diameter. The percent change on diameter for each parameter, respectively, is shown below in Figure 9.



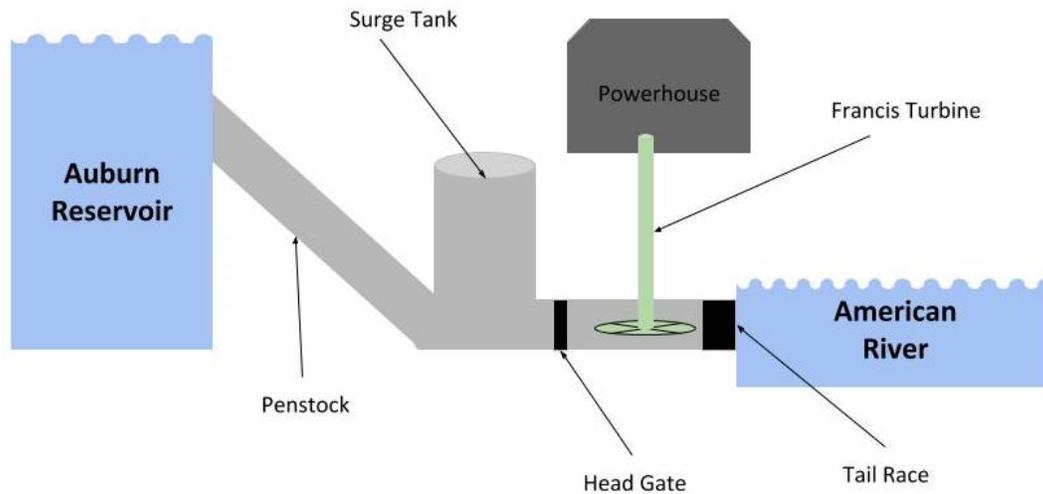
**Figure 9: Sensitivity analysis results of percent change of three parameters effect on penstock pipe diameter.**

These calculations allowed for all necessary design parameters and types of components to be determined to provide a recommendation to the client. The minimum penstock pipe diameter was found to be 10.83 feet but was increased to 11.5 feet for the design to allow for a factor of safety. The surge tank diameter was then 46 feet based on the relationship defined in the application section. This then plugged into an equation that determined the necessary surge tank height ( $H_s$ ) to be 63.7 feet. This was increased to 70 feet to allow for a factor of safety. Equations for this analysis are available in Appendix 1. Table 3 defines the design parameters of the system.

**Table 3: Design parameters for the system**

Parameter	Description	Value (if set)	Units
L	Length of the penstock	580	ft
D	Diameter of penstock	11.5	ft
$D_s$	Diameter of the surge tank	46	ft
$H_s$	Height of the surge tank	70	ft

This design information was then used to create a river outlet power generation system diagram for the Auburn Dam. This diagram serves as an outline for the system analyzed and provides a general layout for the system and is available below in Figure 10.



**Figure 10: Design plan for the system based on the parameters in Tables 1 and 3.**

## Discussion

Using the two specifications provided by the client, a maximum flow rate of 5200 cfs and minimum power generation of 4MW, design parameters were determined and a turbine type was determined. Along with these two key results, other warnings and recommendations were discovered such as the risk of turbulent flow and the sensitivities of key parameters. All this information led to the recommended design parameters and plan for a functional river outlet system for power generation.

The approach taken to solve this problem was to identify the problem, develop the design statement, choose a numerical method, employ the numerical method with Fortran, run and assess the model, analyze the results, and present results to the client. This approach allowed for results to be determined for all design parameters which led to the recommendation of the Francis turbine for the system.

Initially, it was expected that the design would simply be a penstock pipe and a Kaplan turbine to meet the needs of the client. Upon further analysis it was discovered that the high flow rates caused very high Reynold's numbers throughout the system which can cause damage to turbines and pipes. In natural systems, the introduction of sediment into a turbulent system can cause the erosion of Kaplan turbine propellers. The Francis turbine performed better and had a higher energy production rate for a range of flows.

The Francis Turbine was chosen for this system because of its robustness in ranges of both operating elevation head and flow rates. The addition of the head gate and surge tank into the system means that flow can be varied at the discretion of the client so the turbine needed to be able to accommodate such changes in velocity and pressure.

The factor of safety added to the surge tank diameter, penstock diameter, and surge tank height was a way to ensure that the system would function as requested by the client. The calculations done to attain the initial values for these parameters was the minimum requirement based on the max flow rate of 5200 cfs and facility power generation requirement of 4 MW. If flow were to be increased and only minimum specifications were met, catastrophic failure could occur.

The sensitivity analysis was done on three chosen parameters of the system: 1) Length of the penstock, 2) water surface elevation height, and 3) Flow rate. Each parameter was changed from -20% to 20% by increments of 5%. The values were then ran through the model to observe the effect on the penstock diameter. The analysis showed that surface water elevation had the largest effect on the penstock diameter with observed percent changes ranging from -7% to 15%. This means that if the system is operated when surface elevation is larger or smaller than the inactive storage elevation, than the minimum pipe diameter requirement changes. More information on these values is available in Appendix 3.

## Conclusions

The analysis showed that an operational system could be achieved through the implementation of a five component river outlet system that consisted of: 1) Penstock, 2) Head gate, 3) Surge Tank, 4) Turbine, and 5) Tailrace. The use of all the components as specified by the system in Figure 10 using a Francis turbine will meet the objective. The results showed that the main design parameters were a penstock diameter of 11.5 feet, a surge tank diameter of 46 feet, and a surge tank height of 70 ft.

The objective of the analysis was to design a functioning river outlet system for the Auburn Dam that could sustain flows of up to 5200 cfs as well as generate 4MW of power to serve as the power plant for facility operations. This objective was met by explaining the design process that led to the design of a river outlet system utilizing a Francis turbine with theoretical power capabilities of 144 MW at maximum flow.

**Recommendation:** The recommendation would be to air on the side of safety when implementing the design presented in this report. Increases in diameters and heights act as mitigation tools for irregular circumstances such as natural disasters and catastrophic failure. Another consideration is that high flow rates lead to high Reynold's numbers which can cause damage to both turbines and pipes, especially if

any particulate matter enters the system. Based on the maximum flow rate and minimum power specifications provided by the client, the presented system should serve as an effective solution.

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## Appendices

### Appendix 1: Technical Methods and Application

#### Methodology

The method implemented to solve the problem was to develop a Fortran program to deploy the secant method and perform the calculations needed to find the solution. The solution for this problem was the diameter of the penstock pipe in feet.

#### Problem Formulation

The problem was formulated with all parameters and initial conditions were established before the approach was determined. The function within the secant method contained the Bernoulli equation, the Darcy-Weisbach equation, and the Reynold's number formula all to define the parts of the Colebrook equation to employ the secant method subroutine.

Bernoulli Equation: The Bernoulli equation below in equation 2 was defined as point 1 being at the top of the water surface elevation and point 2 being placed at the top of the river elevation downstream of the dam (White 2008).

$$\frac{P_1}{\gamma} + \frac{V_1}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2}{2g} + Z_2 + h_f + h_t + \Sigma h_m \quad (2)$$

Where	$P_i$	is the pressure at point i (lbf/ft <sup>3</sup> )
	$g$	is the Gravitational Constant (32.2 ft/s <sup>2</sup> )
	$V_i$	is the velocity at point i (ft/s)
	$\gamma$	is the specific weight of water (62.4 lbf/ft <sup>3</sup> )
	$Z_i$	is the elevation at point i
	$h_f$	is the fiction head (ft)
	$h_t$	is the head of the turbine (ft)
	$h_m$	is the minor loss from the system (ft)

This equation was simplified by using the large reservoir assumption. The large reservoir assumption assumes that the reservoir of water is large enough that the depth isn't changing and such that there is no velocity at that point. The pressure is atmospheric and the same on both sides of the equation. Minor

losses were also assumed negligible. The simplified form of the Bernoulli equation is available in Equation 3. The goal of this equation is to solve for  $h_f$ .

$$h_f = Z_1 - Z_2 - h_t \quad (3)$$

Darcy-Weisbach Equation: The purpose of solving for  $h_f$  was to calculate a value for  $f$ , the friction factor. This was accomplished by employing the Darcy-Weisbach equation. The general form of the Darcy-Weisbach equation is available below in Equation 4.

$$h_f = f * \frac{L}{D} * \frac{V^2}{2g} \quad (4)$$

Where  $f$  is the friction factor  
 $D$  is the diameter of the pipe (ft)  
 $V$  is the velocity through the pipe(ft/s)  
 $L$  is the length of the pipe (ft)

Reynolds Number Formula: The last piece to be solved for before the Colebrook equation could be used to solve for the diameter was the Reynolds number. The Reynold's number is a way that flow is defined through a pipe. The three types of flows are laminar ( $Re < 2300$ ), transitional ( $2300 < Re < 4000$ ), and turbulent ( $Re > 4000$ ). The Reynold's number formula is shown in Equation 5.

$$Re = \frac{V * D}{\nu} \quad (5)$$

Where  $Re$  is the Reynold's number  
 $D$  is the diameter of the pipe (ft)  
 $V$  is the velocity through the pipe(ft/s)  
 $\nu$  is the kinematic viscosity of water (ft<sup>2</sup>/s)

Colebrook Equation: Given equations 3, 4, and 5 the Colebrook equation could be employed as the final equation to solve for the diameter of the penstock. The Colebrook equation utilized in the secant method of the Fortran program is available below in Equation 6.

$$\frac{1}{\sqrt{f}} = -2.0 * \text{Log}\left(\frac{\epsilon}{3.7D} + \frac{2.51}{Re\sqrt{f}}\right) \quad (6)$$

Theoretical Power Output: Theoretical power output was solved using the hydropower calculator formula available on engineering toolbox. This formula is available below Equation 7 (ENGR Toolbox 2019).

$$P = \rho qgh$$

Where  $P$  power theoretical available  
 $\rho$  density of water  
 $q$  is the volumetric flow of water  
 $h$  is the hydraulic elevation head

## Numerical Methods

The numerical method chosen to solve the problem was the

The secant method. The secant method is a root finding method that is implemented when dealing with more computationally difficult functions. The benefit of the secant method for these inconvenient functions is that a root estimate can be achieved without having to compute a derivative (Chapra and Canale 2008). Given two initial estimates, the secant method iterates until finding an estimate that is within three stopping criteria: 1) tolerance, 2) convergence, 3) maximum iterations. The methods equation is below in Equation 7.

$$x_{i+1} = x_i - \frac{f(x_i)(x_{i-1}-x_i)}{f(x_{i-1})-f(x_i)} \quad (7)$$

Where  $x_i$  is the old estimate  
 $x_{i-1}$  is the older estimate  
 $x_{i+1}$  is the new estimate

## Appendix 2: Raw Code

### Module Types

```
Integer, Parameter:: wp=selected_real_kind(15)
```

```
Real(wp), Parameter:: pi=3.141592653589793238462_wp
```

```
End Module Types
```

### Module Roots

```
Contains
```

```
Subroutine Secant(func, Xolder, Xold, Xnew, T, T1, iter,Flag)
```

Use Types

Implicit None

Real(wp), Intent(inout)::Xold !First Guess (xk)

Real(wp), Intent(inout)::Xolder !previous Guess (xki-1)

Real(wp), Intent(inout)::Xnew !next guess (xk+1)

Real(wp), Intent(in)::T, T1 !Tolerance 1 and 2

Integer, Intent(in)::iter !Iterations

Integer::counter !Counter

Integer, Intent(out)::Flag !Error Flag

Real(wp)::fXolder, fXold, fXnew

!Interfaces in external dummy function

Interface

Function func(d)

Use Types

Real(wp), Intent(in)::d

Real(wp)::func

End Function func

End interface

fXolder=func(Xolder)

!Write(\*,\*) fXk

fXold=func(Xold)

!Write(\*,\*) fXk1

counter=0.D0

Flag=0.D0

!Algorithm

```

Do
    counter=counter+1
    If (counter==iter) Then
        Flag=1
        Return
    End If
    !Defines new Value
    !Flags are the different error checks

    Xnew=Xold-(fXold)*((Xold-Xolder)/(fXold-fXolder))
    fXnew=func(Xnew)
    If (ABS(fXnew)<=T) Then
        Flag=2
        Return
    Else if (fXnew==0) Then
        Flag=0
        Return
    Else If (ABS(Xnew-Xold)<=T1) Then
        Flag=3
        Return
    End If
    Xold=Xolder
    fXold=fXolder
    Xolder=Xnew
    fXolder=fXnew
    !Write(*,*) fXk2
End Do

```

End Subroutine Secant

End Module Roots

Module Var

Use Types

Real(wp) :: g=32.2\_wp !Gravitational Constant (ft/s^2)

Real(wp) :: elev1=816.5\_wp !Water elevation of Reservoir, constant (Feet)

Real(wp) :: elev2=490.5\_wp !elevation of river downstream, constant (Feet)

Real(wp) :: f ! friction factor (Unitless)

Real(wp) :: L= 580.\_wp !(Feet)

Real(wp) :: Q=5200\_wp !(ft^3/sec)

Real(wp) :: v !velocity

Real(wp) :: e=0.00015\_wp !roughness of commercial steel, feet

Real(wp) :: visc=1.2075E-5\_wp !Viscosity of water at 60 degrees f, ft^2/s

Real(wp):: Re !=2300 !Reynolds number

Real(wp):: Ht=303.51\_wp !Head of turbine in feet

End Module Var

Function orifice (d)

Use Types

Use Var

Real(wp), Intent(in)::d

Real(wp)::orifice

$v=Q/(\pi/4*d^{**2})$

$Re= v*d/visc$

!Write(\*,\*) Re

$hf=elev1-elev2-Ht$

```

!Write(*,*) hf
f=hf/((L/d)*(v**2/(2*g)))
!Write(*,*) f
orifice= (1/dSQRT(f))+(2*LOG10((e/(3.7*d))+(2.51/(Re*dSQRT(f))))))
End Function orifice
Program projsecant
Use Types
Use Roots
Use Var
Implicit None
Real(wp):: diameter
Real(wp)::x1=10.5,x2=11,x3 !xolder,xold,xnew
Real(wp):: Tol1=0.001 !Tolerance 1
Real(wp):: Tol2=0.00000001 !Tolerance 2
integer :: numit=1000
Integer :: flagexit
Interface
Function orifice (d)
Use Types
Use Var
Real(wp),Intent(in)::d
Real(wp):: orifice
End Function orifice
End interface
Call Secant(orifice, x1,x2,x3, Tol1, Tol2, numit, flagexit)
Write(*,*) "Pipe diameter is", x3

```

!Write(\*,\*) "Exit Flag is", flagexit

Write(\*,\*) "END PROGRAM"

End Program projsecant

```
#[K#]0;bf151@vlinux-node1: ~/Fortran/ENGR326Labs/secant_2##[01;32mbf151@vlinux-  
node1#[00m:#[01;34m~/Fortran/ENGR326Labs/secant_2#[00m$ gfortran secant_2.f90 -o  
pro#####[1P#[C#[C##[1P##[1P#[1@c#[C##[1P
```

```
#[0;bf151@vlinux-node1: ~/Fortran/ENGR326Labs/secant_2##[01;32mbf151@vlinux-  
node1#[00m:#[01;34m~/Fortran/ENGR326Labs/secant_2#[00m$ pro
```

Pipe diameter is 10.829231295436037

END PROGRAM

### Appendix 3: Raw Data

Appendix 3 contains all the raw data from the analysis of this design project.

**Table 4: Diameter and Flow rate data**

Q (cfs)	D(ft)
5200	10.83
4700	10.41
4200	9.97
3700	9.49
3200	8.98
2700	8.41
2200	7.77
1700	7.03

**Table 5: Sensitivity analysis raw data for the three analyzed parameters**

% Change in Parameter	% Change d for L	% Change d water elevation	% Change d for Flow
20	3.60	-7.50	7.29
15	2.77	-5.91	5.54
10	1.85	-3.88	3.69
5	0.92	-1.85	1.85
0	0.00	0.00	0.00
-5	-1.02	3.05	-1.94

-10	-2.03	6.28	-3.97
-15	-3.14	10.25	-6.09
-20	-4.25	15.05	-8.31

**Table 6: Theoretical power output and Reynold's number vs flow raw data**

Q (cfs)	Power (MW)	Re
5200	144	78330028
4700	130	70798295
4200	116	63266561
3700	102	55734828
3200	88	48203094
2700	75	40671361
2200	61	33139627
1700	47	25607894
1200	33	18076160
700	19	10544427
200	6	3012693
100	3	1506347

**Table 7: Raw data for the auburn dam system provided by the USBR (Reference)**

Project Location: North and Middle Forks of American River, in Placer and El Dorado Counties, near Auburn, California																		
Project Purposes: Irrigation Water Supply, Municipal & Industrial Water Supply, Flood Control, Power, Recreation, Fish & Wildlife, Navigation																		
Drainage Areas		Unimpaired Flows of Auburn Dam																
Auburn Dam (RM 20.1) <sup>[1]</sup>	970 square miles	Mean annual runoff (WYs 1922-1994) <sup>[4]</sup>	1,363,000 acre-feet															
North Fork American R. at Auburn Dam <sup>[1]</sup>	355 square miles	Maximum annual runoff (1982 WY) <sup>[6]</sup>	3,256,000 acre-feet															
N. Shirltail Cyn. Cr. at Sugarpine Dam <sup>[2]</sup>	9 square miles	Minimum annual runoff (1977 WY) <sup>[4]</sup>	229,000 acre-feet															
Middle Fork American R. at North Fork <sup>[1]</sup> (excluding Rubicon River)	300 square miles	Spillway design flood <sup>[5]</sup>																
MF American R. at Fr. Meadows Dam <sup>[2]</sup>	47 square miles	Peak inflow	500,000 cfs															
Rubicon River at MF American River <sup>[1]</sup>	316 square miles	1-day volume	758,000 acre-feet															
Rubicon River at Hell Hole Dam <sup>[2]</sup>	112 square miles	5-day volume	1,700,000 acre-feet															
Pilot Creek at Stumpy Meadows Dam <sup>[2]</sup>	15 square miles	Standard Project Flood <sup>[2]</sup>																
Gerle Creek at Loon Lake Dam <sup>[2]</sup>	8 square miles	Peak Inflow	306,000 cfs															
American River at Folsom Dam <sup>[3]</sup>	1,875 square miles	100-year flood																
American River at Fair Oaks <sup>[4]</sup>	1,921 square miles	Peak Inflow	202,000 cfs															
American River at H Street Bridge <sup>[4]</sup>	1,969 square miles	5-day volume	783,000 acre-feet															
Auburn Dam		Auburn Reservoir																
Dam type	Conc curved-gravity (CG-3)	Elevations																
Location (North Fork American River)	River Mile 20.1	Top of dead storage	616.5 feet msl															
Elevation, top of parapet	1,139.5 feet msl	Top of inactive	816.5 feet msl															
Elevation, crest of dam	1,135.0 feet msl	Top of active conservation	1,083.1 feet msl															
Structural height	685 feet	Top of joint use (gross pool)	1,131.4 feet msl															
Total length of crest	4,150 feet	Area																
Width of crest at elevation 1135.0	40 feet	Gross pool	10,050 acres															
Maximum base thickness	465 feet	Storage capacity																
Downstream face slope	0.68:1	Top of dead storage	29,000 acre-feet															
Total concrete in dam	9,760,000 yd <sup>3</sup>	Top of inactive	380,000 acre-feet															
Diversion tunnel diameter (horseshoe)	33 feet	Top of active conservation	1,876,000 acre-feet															
		Top of joint use (gross pool)	2,326,000 acre-feet															
		Length of shoreline	140 mi															
Spillway (service and auxiliary)		Powerplant																
Crest elevation	980 feet msl	Number and size of units	4 @ 200 MW															
Discharge capacity at maximum water level	330,000 cfs	Type of turbines	Francis															
Total orifice area	3,648 ft <sup>2</sup>	Discharge at rated speed & head	5,760 cfs															
Crest gates (top-seal radial)		Type of generators	vertical shaft															
Number and size	8 @ 19x24 feet	Number and diameter of penstocks	4 @ 17 feet															
Plunge pool basin elev (service / auxiliary)	410 / 430 feet msl	Penstock intake elevations	625 and 800 feet msl															
Outlets		Other Project Features																
River outlets (72-in dia. w/ 72-in ring-follower gates & hollow jet valves)		Major relocations <sup>[7]</sup>	Highway 49, upstr. access roads															
Number and intake elevation	2 @ 625 feet msl	Takeline lands <sup>[7]</sup>	43,473 acres															
Discharge elevation	485.5 feet msl																	
Capacity at top of inactive	4,000 cfs																	
Capacity at gross pool / restr. capacity <sup>[6]</sup>	5,540 cfs / 4,200 cfs																	
<p>Key:</p> <table border="0"> <tr> <td>cfs – cubic feet per second</td> <td>in – inches</td> <td>R – River</td> </tr> <tr> <td>Cr – Creek</td> <td>MF Middle Fork</td> <td>WY – water year</td> </tr> <tr> <td>Cyn – Canyon</td> <td>msl – above mean sea level</td> <td>yd<sup>3</sup> – cubic yard</td> </tr> <tr> <td>dia – diameter</td> <td>MW – megawatt</td> <td></td> </tr> <tr> <td>Fr – French</td> <td>N – North</td> <td></td> </tr> </table>				cfs – cubic feet per second	in – inches	R – River	Cr – Creek	MF Middle Fork	WY – water year	Cyn – Canyon	msl – above mean sea level	yd <sup>3</sup> – cubic yard	dia – diameter	MW – megawatt		Fr – French	N – North	
cfs – cubic feet per second	in – inches	R – River																
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dia – diameter	MW – megawatt																	
Fr – French	N – North																	
<p>Notes:</p> <p>All information presented in Table 1 taken from <i>Feasibility Design Summary, Auburn Dam Concrete Curved-Gravity Dam (CG-3)</i> (US Dept. of the Interior, Water and Power Resources Service, August 1980) unless otherwise noted.</p> <p>[1] California Watershed Map, CALWATER Version 2.2, September 1999, <a href="http://qls.ca.gov/">http://qls.ca.gov/</a></p> <p>[2] <i>Design and Analysis of Auburn Dam Volume One</i>, Reclamation, August 1977</p> <p>[3] <i>Reservoir Regulation Manual for Flood Control, Folsom Dam and Reservoir, Appendix II</i>, U.S. Army District, Corps of Engineers, March 1959</p> <p>[4] Auburn annual inflow data from CALSIM II (CVP OCAP Study 5, June 2004)</p> <p>[5] <i>Auburn Dam site Inflow Spillway Design Flood Study</i>, Reclamation, January 1967</p> <p>[6] Restricted to a discharge of 4,200 cfs because of possible damages to the conduits caused by high-velocity flow</p> <p>[7] <i>Final Report on the Evaluation of the Auburn Dam Project</i>, Bechtel National, Inc., November 1985</p>																		

## Appendix 4: Design Brief

### Wolf Introduction Effects on Lassen National Park

#### Client

The client for this project is both the National Park Service and the California Department of Fish and Game.

#### Design Statement

Design and develop an analysis of the possible effects of the reintroduction of gray wolves into Lassen National Park that allows for future considerations for park expansion projects.

#### Constraints

- 1) Seasonal Constraint: People can only access the roads of the park for a few months out of the year
- 2) Biological Constraint: Very few adult wolves at the moment which restricts model
- 3) Computational Constraint: Not all prey can be analyzed realistically so a specific few will be focused on for this analysis

#### Literature Review

Using numerical methods to solve ordinary differential equations based on predator prey models is a commonly used practice to determine how many predatory tags to release and keep the public safe.

“Numerical solution of Lotka Volterra prey predator model by using Runge–Kutta–Fehlberg method and Laplace Adomian decomposition method” shows a way to accomplish such analysis for these types of models (Mondal, 2016).

#### Methods

This analysis will be done using the Lotka Volterra equations that are famous for their use in creating predator and prey models. These equations will be treated as initial value problems with data coming from the California department of fish and game as well as the potential to use data from highly populated gray wolf states for comparisons. The Numerical method employed will be Runge–Kutta–Fehlberg method that is designed to solve initial value ordinary differential equations. Supplementary analysis will be done with simple population growth models

#### Discussion

This project will look into the newly discovered gray wolf pack that has started in Lassen national park. The presence of wolves has currently been deemed not a threat for people in the area but this is only due to the low population currently in the area. What if this population was to increase? Wolves are very



<<https://www.wildlife.ca.gov/Conservation/Mammals/Deer/Population#32712445-population-by-hunt-zone>> (Feb. 16, 2019).

Paul, S., Mondal, S. P., and Bhattacharya, P. (2016). “Numerical solution of Lotka Volterra prey predator model by using Runge–Kutta–Fehlberg method and Laplace Adomian decomposition method.” *Alexandria Engineering Journal*, 55(1), 613–617.

## Appendix 5: Copy of Technical Memo to Client

### **Interoffice Memorandum**

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To: City of Sacramento

From: Brayden Leach

Subject: Folsom dam safety

Date: March 4, 2019

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### **Client Description**

The client for this project is the city of Sacramento and is based around the necessity to insure the safety of its citizens on all fronts.

### **Background**

The city of Sacramento is the capital of the largest population state in the United States. The city was established during the gold rush era when John Sutter arrived in the area in 1838 founding Sutter’s Fort (“History” 2018). The city was really beginning to establish itself in 1850 and by 1879 was named the capital of California by the Sacramento constitutional convention. The city exists at the fork of the American and Sacramento River. The Folsom Dam is located northeast of the city upstream of the American River.

### **Needs and Interests**

The City of Sacramento has large interests in public safety. In regards to the Folsom Dam, their interests are mostly based around flood safety, environmental protection of organisms at the delta, and the continued use of the area for both recreation and water storage. There has been times that spillway failure has led to the early upstream travel of salmon from the delta, and other organisms have suffered due to mismanagement of water release downstream.

### **Demographics**

The city of Sacramento has a failure large population with over 500,000 as well as another 48,000 from the city of West Sacramento. The city also has an NBA team called the Sacramento Kings and many renovations being done to revitalize the city with the stadium being moved downtown when it used to be in the neighboring city of Natomas as well as a new enforcement of homeless people in public areas such as Cesar Chavez Park.

## **Purpose**

The purpose of this memo is to inform you, the City of Sacramento, of the safety hazard of the Folsom Dam that feeds the American River northeast of downtown.

## **Summary**

The Folsom Dam feeds the American river that flows into the Sacramento River in downtown Sacramento. This dam turns 63 years old this year which makes it 13 years older than the average life expectancy of dams in the United States. Considering how many thousands of people would be affected by any issues occurring with this dam, an analysis of the hydrostatic forces acting on the dam at different times in the year would be for the benefit of the City of Sacramento. These forces are caused by the pressure of the water acting on the dam on the upstream side. This analysis will be done by implementing a numerical method called Simpson's rule that provided an estimate of the cross sectional area of water acting on the dam and water elevation provides an estimate of the total force acting on the surface of the dam.

## **Discussion**

### **Defining the problem**

The Folsom Dam was constructed in 1948 by the US Army Corps of Engineers and was completed in 1956 (Shykowski and Hardigree n.d.). This makes the dam 63 years old this year and with an average dam life expectancy of dams in the US being 50 years, Folsom Dam is getting to an age that the safety of the citizens along the river and in Sacramento should be in question ("Dams" 2012). A recent structural report released by the American Society of Civil Engineers explained that by the year 2020, 85% of dams in the US will be over 50 years old and require some type of structural renovations to continue to be safe for power generation and water storage ("Dams" 2017). This includes the Folsom Dam which was listed by the national inventory of dams to have a high potential for hazard ("National Inventory of Dams" 2017). That being said, the analysis of the hydrostatic force acting on the dam is important in determining the structural safety of Folsom Dam.

### **Gathering data**

Data to complete this analysis will be gathered from the USGS, the Army Corps of Engineers, and from the California state park service. The data needed for the hydrostatic force analysis is the average depth of the lake for each season, the cross sectional area of water acting on the dam found, the weight of the dam, and the height of the dam.

### **Numerical method and implementation**

Once the necessary data was determined, the problem solving approach was developed. It was decided that the best method for determining the hydrostatic forces acting on the Folsom Dam would be the employment of a numerical method. Numerical methods are analytical tools used to make accurate estimates of complex mathematical equations. The chosen numerical method to solve this problem will be the Simpsons Rule. Information on this numerical method is available in the Technical Appendix. This numerical method will be utilized through the creation of a Fortran program that given a set of parameters will model both the cross sectional area of water in front of the dam while also approximating the total hydrostatic force acting on the dam.

### **Anticipated results**

The anticipated results from this analysis will be that given just the hydrostatic forces of the dam, it will be considered safe for operation. If other factors change however such as spillway integrity, seismic activity, and or erosion, the results could be catastrophic.

### **References**

Chapra, S. and Canale, R. (2015). *Numerical Methods for Engineers: Seventh Edition*. McGraw-Hill Education, New York.

“Dams.” (2012). *Maritime Theater*,  
<<http://web.mit.edu/12.000/www/m2012/finalwebsite/problem/dams.shtml>> (Mar. 5, 2019).

“Dams.” (2017). *ASCE's 2017 Infrastructure Report Card*, American Society of Civil Engineers,  
<<https://www.infrastructurereportcard.org/cat-item/dams/>> (Mar. 5, 2019).

Shykowski, J., and Hardigree, M. (n.d.). *On developing prescriptions for freshwater flows to sustain desirable fishes in the Sacramento-San Joaquin Delta | Center for Watershed Sciences*,  
<[https://watershed.ucdavis.edu/shed/lund/dams/Folsom/FolsomDam.html#General Information](https://watershed.ucdavis.edu/shed/lund/dams/Folsom/FolsomDam.html#General%20Information)> (Mar. 5, 2019).

“National Inventory of Dams.” (2017). *National Inventory of Dams (NID) - Home*, <[https://nid-test.sec.usace.army.mil/ords/f?p=105:113:25127646235555::NO:113,2:P113\\_RECORDID:5410](https://nid-test.sec.usace.army.mil/ords/f?p=105:113:25127646235555::NO:113,2:P113_RECORDID:5410)> (Mar. 5, 2019).

“History.” (2018). *City of Sacramento word treatment*, <<http://www.cityofsacramento.org/fire/about/history>> (Mar. 6, 2019).

## Technical Appendix

### Approach

The approach to solving this problem is the implementation of a numerical method called Simpsons Rule that uses a higher order polynomial to estimate the integral of a given function (Chapra and Canale 2015). The general form of Simpsons Rule is shown below in Equation 1. This numerical method results in a fifth order of error.

$$I \cong (b - a) \frac{f(x_0) + 4f(x_1) + f(x_2)}{6} \quad (1)$$

Where:  $I$  is the approximate integral of the function

$(b - a)$  is the width of the function

$f(x)$  are the values of the function at certain points

The equation used to calculate total force is defined by equation 2.

$$f_t = \int_0^D p g w(z) (D - z) dz \quad (2)$$

Where:  $f_t$  is the total force acting on the dam

$D$  is the elevation of the dam

$p$  is the density of water

$g$  is the gravitational constant

$w(z)$  is the width of the dam face

## Appendix 6: Annotated Bibliography including all of your references

- Ali, M. H., Alam, M. R., Haque, M. N., and Alam, M. J. (2012). "Comparison of Design and Analysis of Concrete Gravity Dam." *Natural Resources*, 03(01), 18–28. Discusses the ways the concrete gravity dams are designed and analyzed among various different styles and regions of the world.
- Georgakakos, A., Yao, H., Kistenmacher, M., Georgakakos, K., Graham, N., Cheng, F.-Y., Spencer, C., and Shamir, E. (2012). "Value of adaptive water resources management in Northern California under climatic variability and change: Reservoir management." *Journal of Hydrology*, 412-413, 34–46. Explains the ways that the water is changing and how water resources management needs to adapt to the effects of climate change to meet the needs of people in northern California.
- Kiersch, G. A., and Treasher, R. C. (1955). "Investigations, areal and engineering geology; Folsom Dam Project, central California." *Economic Geology*, 50(3), 271–310. Geological analysis of the area of the Folsom dam that explain the mitigation tactics for pore pressure and seismic activity. Design plans and analysis for the Folsom dam that include the original blueprints for the spillways system and curved concrete gravity dam structure.
- Petaccia, G., Lai, C., Milazzo, C., and Natale, L. (2016). "The collapse of the Sella Zerbino gravity dam." *Engineering Geology*, 211, 39–49. Explains many different examples of concrete gravity dam catastrophic failure resulting in deaths and destruction. Emphasizes the importance of diligence in engineering design plans for such structures.
- Yazd, H. G. H., Arabshahi, S. J., Tavousi, M., and Alvani, A. (2015). "Optimal Designing of Concrete Gravity Dam using Particle Swarm Optimization Algorithm (PSO)." *Indian Journal of Science and Technology*, 8(12). Provides more information for the designing of concrete gravity dams in different parts of the world such as India. This article also provides an optimization algorithm.