Observer-Based Fuzzy Adaptive Dynamic Surface Control of Uncertain Nonstrict Feedback Systems With Unknown Control Direction and Unknown Dead-Zone

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***Abstract*—In this paper, an observer-based fuzzy adaptive controller for a class of uncertain nonstrict nonlinear systems with unknown control direction and unknown dead-zone is presented. First, by using equivalence dead-zone inverse and a linear state transformation, the original system is converted to a new one. Then, by using fuzzy logic systems, the unknown nonlinearities are approximated based on an adaptive mechanism, and a nonlinear fuzzy state observer is designed to estimate immeasurable states. The dynamic surface control technique is employed to solve the problem of explosion of complexity in the traditional backstepping approach, and then, this method is combined with Nussbaum gain function to address the problem of unknown control direction. Besides, barrier Lyapunov function is employed to overcome the violation of system output. The proposed controller guarantees that the closed-loop system is stable; all the system states are bounded, and tracking errors converge to a neighborhood of the origin. A numerical simulation is provided to confirm the usefulness of the proposed control design.**

***Index Terms*—Barrier Lyapunov function, fuzzy logic, nonstrict feedback systems, Nussbaum functions, observer-based control.**

I. INTRODUCTION

THERE have been significant developments in nonlinear feedback control by using feedback linearization technique in the past decades, but this method cannot handle uncertainties in the system. Therefore, all attentions have been devoted to the methods based on Lyapunov stability theory, i.e., backstepping and sliding mode control approaches [1]–[6]. Backstepping controller is a well-known approach that systematically designs the final control signal via designing virtual control laws at each step [7], [8]. Although the backstepping controller is useful for these systems, the explosion of complexity is a challenging problem in this method. In addition, sliding mode control requires a mathematical background of systems with matched and mismatched uncertainties.

To avoid mathematical difficulties of the explosion of complexity, dynamic surface control (DSC) technique has been employed [9]–[14]. In DSC, differentiators are replaced with low-pass filters and multiple sliding surfaces [9]. For example, in [12], predictor-based neural DSC has been designed for a class of uncertain nonlinear systems in strict feedback form.

In order to handle uncertainties in nonlinear systems, one can employ universal approximators such as neural networks, or fuzzy logic systems (FLSs), which have been recently utilized in several applications, see [15]–[24]. More specifically, in [18], adaptive fuzzy controllers have been investigated for a class of uncertain SISO nonlinear systems. Furthermore, an adaptive fuzzy control for a class of MIMO nonlinear systems with the aid of feedforward/feedback strategy has been proposed in [25].

In practice, the system states are often unavailable for feedback and therefore, and one can use an observer to estimate immeasurable states. Therefore, different structures for observer design of uncertain nonlinear systems have been introduced. For example, adaptive neural control based on estimated states have been presented in [26].

Another point in regards to controlling nonlinear systems is that the control gain may be unknown *in advanc*e and therefore, it is assumed that the control direction is known [26], [27]. Nussbaum gain function is a method that can be useful for designing a controller for systems with unknown control direction. This method was first proposed by Nussbaum [28], and it is effective to approximate the sign of control direction. The Nussbaum function is made of increasing ultimately function and switching functions, where, the switching function can obtain the sign of control direction [26].

Generally speaking, the above-mentioned problems and considerations are made for systems in strict-feedback form. However, this can be considered as a limitation for dynamical systems because many practical systems cannot be represented in strict-feedback form. In fact, this is a new challenge, and

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if the system has a nonstrict feedback form, the controller design is harder because in strict-feedback systems, a virtual control signal α_i and uncertain function f_i are a function of $\bar{x}_i = [x_1, x_2, \ldots, x_i]^T$ for $i = 1, 2, \ldots, n,$. However, in nonstrict feedback systems, α_i and f_i are functions of the state vector $x = [x_1, x_2, \ldots, x_n]^T$, and it makes the problem more challenging [29]–[32]. Therefore, in recent years, there has been great attention to this class of nonlinear systems. For example, in [31], a neuro-adaptive output-feedback controller has been designed for a class of MIMO nonstrict-feedback nonlinear time-delay systems.

In many systems, dead-zone nonlinearity is one of the significant concerns that is a problematic issue and it may exist in many physical systems and affects on the system stability [33]–[36]. However, to the best of the authors' knowledge, no results have been reported for adaptive output feedback fuzzy control of nonstrict feedback systems with unknown dead zone and unknown control direction, which motivate us for the current study. Another concern is the constraints on the states and output of the system. In order to prevent these constraints, one can employ barrier Lyapunov function [37]–[40].

Motivated by the above concerns, this paper presents an observer-based fuzzy adaptive controller based on DSC technique for a class of uncertain nonstrict feedback systems with unmeasured states, unknown control direction, and unknown dead-zone, with constraints on the output. In the proposed controller, by using fuzzy logic approximators, unknown nonlinear functions in nonstrict feedback system are estimated adaptively by using Lyapunov-based adaptation law, and by utilizing fuzzy state observer, the immeasurable states can be estimated. Control direction in this system is unknown, and therefore with the aid of a special Nussbaum function and the barrier Lyapunov function, an appropriate controller is designed. It is assumed that the control signal is passed through a dead-zone function with unknown slopes and the parameters of the dead-zone nonlinearity are approximated by using adaptive mechanism. Finally, by combining the proposed tools and with the aid of DSC, an observer-based adaptive fuzzy controller for nonstrict systems with unknown dead-zone and unknown control direction is designed. It is shown that by using the proposed observer-based fuzzy adaptive controller, the semi-globally uniformly ultimately boundedness (SGUUB) of signals in the closed-loop system is assured and the system output tracks the desired signal effectively with constraints on the output. Moreover, the main contributions of this paper compared to the existing results are summarized as follows.

1) An observer-based fuzzy adaptive controller is proposed using the barrier Lyapunov function for an uncertain nonstrict nonlinear system with unknown control direction and unknown dead zone.
2) By employing FLSs, the unknown nonlinearities are approximated, and a state observer is then designed to estimate immeasurable states.
3) In order to overcome "explosion of terms" which is problematic in traditional backstepping approach, DSC technique is employed, and this technique is combined with Nussbaum gain utilization to overcome the problem of the unknown gain sign.
4) In [12] and [22], the unknown nonlinear systems are in strict feedback form. However, in this paper, an adaptive controller is designed for an extensive class of nonlinear systems in nonstrict feedback forms.
5) Although there are numerous results on control nonlinear systems in the presence of dead-zone (see [35], [36], and [41]), observer-based adaptive fuzzy controller for nonstrict-feedback nonlinear systems with dead-zone, and unknown control direction has not been investigated yet.

In what follows, the mathematical preliminaries are given in Section II. The system dynamic and problem formulation are presented and a fuzzy observer is introduced in Section III. Section IV explains fuzzy adaptive control design of nonstrict systems and results on the stability conditions are also discussed in this section. Simulation results are provided in Section V. Finally, Section VI concludes this paper.

II. Mathematical Preliminaries

In this section, the mathematical preliminaries that are used in this paper are given.

A. Nussbaum Function Scheme

Definition 1 [26]: The function $N(\eta)$ is called Nussbaum gain function if the following properties are held:

$$\lim_{s\to\infty} \sup \frac{1}{s}\int_0^s N(\eta)d\eta = +\infty \tag{1}$$

$$\lim_{s\to\infty} \inf \frac{1}{s}\int_0^s N(\eta)d\eta = -\infty. \tag{2}$$

Different functions have been introduced that satisfy the above conditions, e.g., $\exp(\eta^2)\cos([\pi/2]\eta)$, $\eta^2\cos(\eta)$, and $\eta^2\sin(\eta)$. In this paper, $\exp(\eta^2)\cos(\eta^2)$ has been chosen as a Nussbaum function.

Lemma 1 [26]: Let $N(\eta)$ be Nussbaum gain function and η is assumed to be a smooth function defined on $[0, t_f]$. Suppose a positive definite, radially unbounded function $V(t)$ exists that satisfies the following inequality:

$$0 \le V(t) \le V(0) + e^{-Ct}\int_0^t d(\rho N(\eta)+1)\dot{\eta}e^{C\tau}d\tau + D \tag{3}$$

then $V(t)$, $\eta(t)$, and $\int_0^t d(\rho N(\eta)+1)\dot{\eta}e^{C\tau}d\tau$ are bounded on $[0, t_f]$ where C and D are positive constants, ρ is a nonzero constant and d is some suitable constant.

B. Barrier Lyapunov Function

Definition 2 [40], [42]: A scalar function $V_i(x)$ is a barrier Lyapunov function defined relating to the system $\dot{x}_i = f_i(x_i)$ on an open region E_i including the origin, which is continuous, positive definite, and with continuous first-order partial derivatives at every point of E_i. In addition, as x_i tends to the boundary of E_i results in $V_i(x) \to \infty$. It also holds the property $V_i(x_i(t)) \le b_i\ \forall t \ge 0$ along the solution of $\dot{x}_i = f_i(x_i)$ for

$x_i(0) \in E_i$ and some positive constant b_i. In this paper, barrier Lyapunov function is defined as the following function:

$$\bar{V}_1 = \frac{1}{2}\log\left(\frac{K^2}{K^2 - s_1^2}\right) \tag{4}$$

where $|s_1| < K$ and log(.) stands for the logarithm function of (.).

Lemma 2: For any positive constant K, where s_1 satisfies $|s_1| < K$, we have the following inequality:

$$\bar{V}_1 = \frac{1}{2}\log\left(\frac{K^2}{K^2 - s_1^2}\right) < \frac{s_1^2}{K^2 - s_1^2}. \tag{5}$$

C. Fuzzy Logic Approximator System

The FLS contains four parts. These parts include the knowledge base, the fuzzifier, the fuzzy inference engine working on fuzzy rules, and the defuzzifier [13]. The knowledge base is a set of If–Then rules in the following form: $\Re^l$: If x_1 is F_1^L and x_2 is F_2^L and $\cdots$ and x_n is F_n^L, Then y is G^L, $L = 1, 2, \ldots, N$ where $x = [x_1, x_2, \ldots, x_n]^T$ and y are the inputs and output of the FLSs, respectively, and N is the number of rules.

By using singleton function, center average defuzzification and product inference [23], the FLS can be expressed as follows:

$$y(\mathbf{x}) = \frac{\sum_{L=1}^{N} \bar{y}_L \prod_{i=1}^{n} \mu_{F_i}^L(x_i)}{\sum_{L=1}^{N}\left[\prod_{i=1}^{n} \mu_{F_i}^L(x_i)\right]} \tag{6}$$

where $\mu_{F_i^L}$ and μ_{G^L} are fuzzy membership functions associated with fuzzy sets F_i^L and G^L, and $\bar{y}_L = \max_{y \in \Re} \mu_{G^L}(y)$. Define the basis function in the FLS as follows:

$$\varphi_L(\mathbf{x}) = \frac{\prod_{i=1}^{n} \mu_{F_i}^L(x_i)}{\sum_{L=1}^{N}\left[\prod_{i=1}^{n} \mu_{F_i}^L(x_i)\right]}. \tag{7}$$

Denoting $\theta^T = [\theta_1, \theta_2, \ldots, \theta_N]$ and $\varphi(\mathbf{x}) = [\varphi_1(\mathbf{x}), \ldots, \varphi_N(\mathbf{x})]^T$, FLS (6) can be stated as follows:

$$y(x) = \theta^T \varphi(x). \tag{8}$$

Lemma 3 [43]: Suppose $f(x)$ is a continuous function on a compact set Ω. Then, there exists an FLS (8), for any constant $\varepsilon > 0$ such that

$$\sup_{\mathbf{x} \in \Omega}\left|f(\mathbf{x}) - \theta^T \varphi(\mathbf{x})\right| \le \varepsilon. \tag{9}$$

III. Problem Description

Consider a nonstrict nonlinear system with unknown disturbances, unknown control direction, and unknown dead-zone described as follows:

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(\mathbf{x}) + d_i(t) \ \ i = 1, 2, \ldots, n-1 \\ \dot{x}_n = \rho D(u) + f_n(\mathbf{x}) + d_n(t) \\ y = x_1 \end{cases} \tag{10}$$

where $\mathbf{x} = [x_1, x_2, \ldots, x_n]^T$ is the state vector, y is the output of the system, and ρ is an unknown constant satisfying $\rho_{\min} \le |\rho| \le \rho_{\max}$, where $\rho_{\min}$ and $\rho_{\max}$ are positive constants, $f_i(x)$, $i = 1, 2, \ldots, n$ are unknown smooth nonlinear functions, $d_i(t)$, $i = 1, 2, \ldots, n$ are the dynamic disturbances satisfying $|d_i(t)| \le \bar{d}_i$ where $\bar{d}_i$ are known constants, $D(u)$ is the output of the dead-zone nonlinearity described by the following function:

$$D(u) = \begin{cases} m_r(u - d_r), & \text{if } u \ge d_r \\ 0, & \text{if } -d_l < u < d_r \\ m_l(u + d_l), & \text{if } u \le -d_l \end{cases} \tag{11}$$

In (11), $u \in \Re$ is the input of the dead-zone element, d_r and d_l are the dead-zone widths, m_r and m_l are the slopes of the dead-zone and these parameters are assumed to be unknown.

The main is to design an adaptive fuzzy DSC for nonstrict feedback systems such that the system states are bounded and the output of the system can track the desired signal y_r well.

Assumption 1: The reference signal y_r and its derivative y_r, $\dot{y}_r$, and $\ddot{y}_r$ are assumed to be bounded and available for measurement.

Assumption 2 [23]: The output of dead-zone $D(u)$ is not available for measurement. The parameters of dead-zone m_r, m_l, d_r, and d_l are unknown, but their signs are known ($m_r > 0$, $m_l > 0$, $d_r \ge 0$, and $d_l \le 0$).

Assumption 3 [23]: The slopes of the dead-zone are bounded by known constant, i.e., there exist known constants $m_{r\min}$, $m_{r\max}$, $m_{l\min}$, and $m_{l\max}$ such that $0 < m_{r\min} \le m_r \le m_{r\max}$ and $0 < m_{l\min} \le m_l \le m_{l\max}$.

Remark 1: Assumption 1 is common in the literature since the output of the system and its derivatives are bounded for real systems. Assumption 2 is reasonable for realistic applications, because the actuator in a real system has a predefined range of operation. Therefore, the closed-loop system might be unstable if this limitation is not considered in the design procedure.

If we assume the system is without a dead-zone and u_d is the input of the plant, the following control signal u is generated from the equivalence dead-zone inverse:

$$u = D^{-1}(u_d) = \frac{u_d + \hat{d}_{mr}}{\hat{m}_r}\delta + \frac{u_d - \hat{d}_{ml}}{\hat{m}_l}(1 - \delta). \tag{12}$$

In (12), $\hat{m}_r$, $\hat{m}_l$, $\hat{d}_{mr}$, and $\hat{d}_{ml}$ are estimates of m_r, m_l, d_{mr}, d_{ml} and

$$\delta = \begin{cases} 1 & \text{if } u_d \ge 0 \\ 0 & \text{if } u_d < 0. \end{cases} \tag{13}$$

The resulting error between the u and u_d is obtained as

$$\begin{aligned} D(u) - u_d = &\left(\tilde{d}_{mr} - \frac{u_d + \hat{d}_{mr}}{\hat{m}_r}\tilde{m}_r\right)\delta \\ &+ \left(\tilde{d}_{ml} - \frac{u_d - \hat{d}_{ml}}{\hat{m}_l}\tilde{m}_l\right)(1 - \delta) + \varepsilon_d \end{aligned} \tag{14}$$

where $\tilde{d}_{mr} = d_{mr}^* - \hat{d}_{mr}$, $\tilde{d}_{ml} = d_{ml}^* - \hat{d}_{ml}$, $\tilde{m}_r = m_r^* - \hat{m}_r$, and $\tilde{m}_l = m_l^* - \hat{m}_l$ are parameters of error and $\varepsilon_d = -m_r^* \chi_r(u - d_r^*) - m_l^* \chi_l(u - d_l^*)$ is bounded. In addition

$$\chi_r = \begin{cases} 1, & \text{if } 0 \le u < d_r \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad \chi_l = \begin{cases} 1, & \text{if } d_l < u < 0 \\ 0, & \text{otherwise.} \end{cases} \tag{15}$$

Since the system states may not be available for measurement, an observer should be incorporated to estimate the unmeasured states. In order to design an appropriate observer, at first a suitable change of variable should be employed. Let $\chi_i = (x_i/\rho)$, then system (10) can be represented as

$$\begin{cases} \dot{\chi}_1 = \chi_2 + \frac{F_1(\boldsymbol{\chi})}{\rho} + \frac{d_1(t)}{\rho} \\ \quad\vdots \\ \dot{\chi}_n = D(u) + \frac{F_n(\boldsymbol{\chi})}{\rho} + \frac{d_n(t)}{\rho} \\ y = \rho\chi_1. \end{cases} \tag{16}$$

By defining $\bar{F}_i(\boldsymbol{\chi}) = [F_i(\boldsymbol{\chi})/\rho]$, and $\bar{d}_i(t) = (d_i(t)/\rho)$, $i = 1, 2, \ldots, n$, the system (16) can be rewritten as

$$\begin{cases} \dot{\chi}_1 = \chi_2 + \bar{F}_1(\boldsymbol{\chi}) + \bar{d}_1(t) \\ \quad\vdots \\ \dot{\chi}_n = D(u) + \bar{F}_n(\boldsymbol{\chi}) + \bar{d}_n(t) \\ y = \rho\chi_1. \end{cases} \tag{17}$$

After adding and subtracting new term $K\chi_1$, (17) can be compressed as follows:

$$\begin{aligned} \dot{\boldsymbol{\chi}} &= A\boldsymbol{\chi} + K\chi_1 + \sum_{i=1}^{n} B_i\bar{F}_i(\boldsymbol{\chi}) + \Pi + B_nD(u) \\ y &= C^T\boldsymbol{\chi}, \quad i = 1, 2, \ldots, n \end{aligned} \tag{18}$$

where

$$A = \begin{bmatrix} -k_1 & & & \\ \vdots & & I & \\ -k_n & 0 & \cdots & 0 \end{bmatrix}, \quad K = \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{bmatrix}$$

$$B_i = \begin{bmatrix} \underbrace{0 \ \cdots \ 0 \ 1}_{i} & 0 & \cdots & 0 \end{bmatrix}_{1\times n}^T, \quad \Pi = [d_1, d_2, \ldots, d_n]^T$$

$$C^T = \begin{bmatrix} \rho & 0 & \cdots & 0 \end{bmatrix}, \ \bar{F}_i = \begin{bmatrix} \bar{F}_1, \ldots, \bar{F}_n \end{bmatrix}^T$$

and $\boldsymbol{\chi} = [\chi_1, \chi_2, \ldots, \chi_n]^T$. The vector K is chosen such that matrix A is Hurwitz. It means that there exists a positive definite matrix $P > 0$, for any given positive definite matrix $Q > 0$, such that the following equality holds:

$$A^TP + \mathrm{PA} = -2Q. \tag{19}$$

In (18), $\bar{F}_i(\boldsymbol{\chi})$ is unknown. Therefore, by using Lemma 3, $\bar{F}_i(\boldsymbol{\chi})$, $i = 1, 2, \ldots, n$, can be approximated by the following FLS:

$$\hat{F}_i(\hat{\boldsymbol{\chi}}|\theta_i) = \theta_i^T\varphi_i(\hat{\boldsymbol{\chi}}) \tag{20}$$

where $\hat{\boldsymbol{\chi}} = [\hat{\chi}_1, \ldots, \hat{\chi}_n]^T$ is the estimate of $\boldsymbol{\chi} = [\chi_1, \chi_2, \ldots, \chi_n]^T$. The optimal parameter vector θ_i^* is defined as follows:

$$\theta_i^* = \arg\min_{\theta_i\in\Omega_i}\left[\sup_{\boldsymbol{\chi}\in U, \hat{\boldsymbol{\chi}}\in\hat{U}}\left|\hat{F}_i(\hat{\boldsymbol{\chi}}|\theta_i) - \bar{F}_i(\hat{\boldsymbol{\chi}})\right|\right]. \tag{21}$$

In (21), Ω_i, U, and $\hat{U}$ are the compact regions for θ_i, $\boldsymbol{\chi}$, and $\hat{\boldsymbol{\chi}}$, respectively. The approximation errors δ_i and FLS minimum approximation errors ε_i can be defined as

$$\begin{aligned} &\delta_i = \bar{F}_i(\boldsymbol{\chi}) - \hat{F}_i(\hat{\boldsymbol{\chi}}|\theta_i), \ \ \varepsilon_i = \bar{F}_i(\boldsymbol{\chi}) - \hat{F}_i(\hat{\boldsymbol{\chi}}|\theta_i^*) \\ &\quad i = 1, \ldots, n. \end{aligned} \tag{22}$$

In (22), $|\delta_i| \le \delta_i^*$ and $|\varepsilon_i| \le \varepsilon_i^*$ that δ_i^* and ε_i^* are known positive constants. By substituting (22) into (18), it is rewritten as

$$\begin{aligned} \dot{\boldsymbol{\chi}} &= A\boldsymbol{\chi} + K\chi_1 + \sum_{i=1}^{n} B_i\Big(\hat{F}_i(\hat{\boldsymbol{\chi}}|\theta_i) + \delta_i\Big) + \Pi + B_nD(u) \\ y &= C^T\boldsymbol{\chi}, \ \ i = 1, 2, \ldots, n. \end{aligned} \tag{23}$$

Therefore, the following observer is designed:

$$\begin{aligned} \dot{\hat{\boldsymbol{\chi}}} &= A\hat{\boldsymbol{\chi}} + \sum_{i=1}^{n} B_i\hat{F}_i(\hat{\boldsymbol{\chi}}|\theta_i) + B_nD(u) \\ \hat{y} &= C^T\hat{\boldsymbol{\chi}}, \ \ i = 1, 2, \ldots, n. \end{aligned} \tag{24}$$

The observation error vector e is defined as

$$e = [e_1, e_2, \ldots, e_n]^T = \boldsymbol{\chi} - \hat{\boldsymbol{\chi}}. \tag{25}$$

By using (23), (24), and observation error (25), the following observer error dynamic is obtained:

$$\dot{e} = Ae + K\chi_1 + \sum_{i=1}^{n} B_i\delta_i + \Pi \tag{26}$$

where $\delta_i = [\delta_1, \delta_2, \ldots, \delta_n]^T$.

In this part, the Lyapunov function $V_0 = (1/2)e^TPe$ is introduced, where $P = P^T > 0$ is a positive definite matrix, and by utilizing (19) and (26), we have

$$\begin{aligned} \dot{V}_0 &= \mathrm{sym}\left(\frac{1}{2}(Ae + \delta + \Pi + K\chi_1)^TPe\right) \\ &= \frac{1}{2}e^T\underbrace{\left(A^TP + \mathrm{PA}\right)}_{-2Q}e + e^TP(\delta + \Pi) + e^TPK\chi_1 \end{aligned} \tag{27}$$

where sym($*$) denotes $(*) + (*)^T$.

In addition, the following inequality is true for a positive definite matrix Q:

$$\lambda_{\min}(Q)\|e\|^2 \le e^TQe \le \lambda_{\max}(Q)\|e\|^2. \tag{28}$$

By using Young's inequalities, one can obtain

$$e^TP(\Pi + \delta) \le \|e\|^2 + \frac{\|P\|^2}{2}\left\|\delta^*\right\|^2 + \frac{\|P\|^2}{2}\sum\bar{d}_i^2. \tag{29}$$

By using inequalities (28) and (29), (27) can be expressed as follows:

$$\dot{V}_0 \le -(\lambda_{\min}(Q) - 1)\|e\|^2 + M_1 + e^TPK\chi_1 \tag{30}$$

where $M_1 = (1/2)\|P\|^2\|\delta^*\|^2 + (1/2)\|P\|^2\sum_{i=1}^{n}\bar{d}_i^2$, that M_1 is a positive constant. It is noticeable that the last inequality (30) is used later for stability proof of the designed controller.

IV. Adaptive Fuzzy Control Design and Stability Analysis

In this section, by using an adaptive fuzzy scheme and employing DSC approach, a suitable controller with adaptive laws is designed. It will be shown that the SGUUB of all the signals of the closed-loop system is achieved, and also the output of the system tracks the reference signal. In order to

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design DSC, it is necessary to recursively follow the procedure in n-step. At first, the following change of coordinate is considered:

$$\begin{aligned} s_1 &= y - y_r \\ s_i &= \hat{\chi}_i - z_i,\ \ i = 2, 3, \ldots, n \\ w_i &= z_i - \alpha_{i-1} \end{aligned} \quad (31)$$

where s_i is the error surface, y_r is the desired signal for tracking, α_{i-1} are the virtual control, z_i is a state variable that it is achieved by using a first-order filter on control function α_{i-1} and w_i is the output error of this filter.

Step 1: From (31), we get

$$\dot{s}_1 = \dot{y} - \dot{y}_r = \rho\chi_2 + F_1(\boldsymbol{\chi}) + d_1 - \dot{y}_r. \quad (32)$$

By substituting (21) and (22) in (32), and by defining the error $e_2 = \chi_2 - \hat{\chi}_2$, $s_2 = \hat{\chi}_2 - z_2$, and $w_2 = z_2 - \alpha_1$ one can obtain

$$\begin{aligned} \dot{s}_1 &= \dot{y} - \dot{y}_r = \rho\chi_2 + F_1(\boldsymbol{\chi}) + d_1 - \dot{y}_r \\ &= \rho\left(e_2 + \hat{\chi}_2\right) + \rho\varepsilon_1 + \rho\theta_1^{*^T}\varphi_1(\hat{\boldsymbol{\chi}}) + d_1 - \dot{y}_r \\ &= \rho\left(e_2 + \hat{\chi}_2 + \varepsilon_1\right) + \underbrace{\rho\tilde{\theta}_1^T}_{\tilde{\theta}_{g1}^T}\varphi_1\left(\boldsymbol{\chi}\right) + \underbrace{\rho\theta_1^T}_{\theta_{g1}^T}\varphi_1\left(\hat{\boldsymbol{\chi}}\right) + d_1 - \dot{y}_r \\ &= \rho(e_2 + s_2 + w_2 + \alpha_1 + \varepsilon_1) + \underbrace{\rho\tilde{\theta}_1^T}_{\tilde{\theta}_{g1}^T}\varphi_1\left(\hat{\boldsymbol{\chi}}\right) \\ &\quad + \underbrace{\rho\theta_1^T}_{\theta_{g1}^T}\varphi_1\left(\hat{\boldsymbol{\chi}}\right) + d_1 - \dot{y}_r \end{aligned} \quad (33)$$

where $\tilde{\theta} = \theta^* - \theta$ and $\boldsymbol{\chi} = [\chi_1, \chi_2, \ldots, \chi_n]^T$.

Consider the following Lyapunov function candidate:

$$\begin{aligned} V_1 &= V_0 + \bar{V}_1 + \frac{1}{2\gamma_1}\tilde{\theta}_{g1}^T\tilde{\theta}_{g1} \\ &= V_0 + \frac{1}{2}\log\frac{K^2}{K^2 - s_1^2} + \frac{1}{2\gamma_1}\tilde{\theta}_{g1}^T\tilde{\theta}_{g1} \end{aligned} \quad (34)$$

where γ_1 and K are the positive design constants, and $\bar{V}_1$ is a barrier Lyapunov function defined in (4).

By calculating the time derivative of V_1, and using the previous results we have

$$\begin{aligned} \dot{V}_1 &\le -(\lambda_{\min}(Q) - 1)\|e\|^2 + M_1 + e^T PK\chi_1 + \frac{s_1\dot{s}_1}{K^2 - s_1^2} \\ &\quad + \frac{1}{\gamma_1}\tilde{\theta}_{g1}^T\dot{\tilde{\theta}}_{g1} \le -(\lambda_{\min}(Q) - 1)\|e\|^2 \\ &\quad + \frac{s_1}{K^2 - s_1^2}\left(\rho\left(\underbrace{e_2 + \varepsilon_1}_{\{1\}} + \underbrace{s_2 + w_2}_{\{2\}} + \alpha_1\right) + \tilde{\theta}_{g1}^T\varphi_1\left(\hat{\boldsymbol{\chi}}\right)\right. \\ &\quad \left. + \theta_{g1}^T\varphi_1\left(\hat{\boldsymbol{\chi}}\right) + \underbrace{d_1}_{\{3\}} - \dot{y}_r\right) \\ &\quad + \frac{1}{\gamma_1}\tilde{\theta}_{g1}^T\dot{\tilde{\theta}}_{g1} + \underbrace{e^T PK\chi_1}_{\{4\}} + M_1. \end{aligned} \quad (35)$$

In addition, by using the Young's inequalities, the following inequality is true corresponding to the part including term $\{1\}$:

$$\begin{aligned} \frac{s_1}{K^2 - s_1^2}\rho e_2 + \frac{s_1}{K^2 - s_1^2}\rho\varepsilon_1 &\le \frac{1}{2}\|e\|^2 + \frac{1}{2}\frac{s_1^2}{\left(K^2 - s_1^2\right)^2}\rho_{\max}^2 \\ &\quad + \frac{1}{2}\frac{s_1^2}{\left(K^2 - s_1^2\right)^2} + \frac{1}{2}\rho_{\max}^2\left\|\varepsilon_1^*\right\|^2. \end{aligned} \quad (36)$$

In addition, the same expression can be stated for term which includes $\{2\}$ as

$$\begin{aligned} \frac{s_1}{K^2 - s_1^2}\rho s_2 + \frac{s_1}{K^2 - s_1^2}\rho w_2 &\le \frac{s_1^2}{2\left(K^2 - s_1^2\right)^2} + \frac{1}{2}\rho_{\max}^2 s_2^2 \\ &\quad + \frac{s_1^2}{2\left(K^2 - s_1^2\right)^2}\rho_{\max}^2 + \frac{1}{2}w_2^2. \end{aligned} \quad (37)$$

Using the same fashion, we may write the following inequalities for managing terms including $\{3\}$ and $\{4\}$ in (35)

$$\frac{s_1}{K^2 - s_1^2}d_1 \le \frac{s_1^2}{2\left(K^2 - s_1^2\right)} + \frac{1}{2}\left\|\bar{d}_1\right\|^2 \quad (38)$$

$$\begin{aligned} e^T PK\chi_1 &\le \frac{1}{2}\|e\|^2\|P\|^2 + \frac{1}{2}\frac{\|K\|^2 y^2}{\rho_{\min}} \le \frac{1}{2}\|e\|^2\|P\|^2 \\ &\quad + \frac{\|K\|^2 s_1^2}{2\rho_{\min}} + \frac{\|K\|^2 y_r^2}{2\rho_{\min}^2} + \frac{\|K\|^4 y_r^2}{2\rho_{\min}^4} + \frac{1}{2}s_1^2. \end{aligned} \quad (39)$$

Substituting (36)–(39) into (35), yields

$$\begin{aligned} \dot{V}_1 &\le -(\lambda_{\min}(Q) - 1)\|e\|^2 + M_1 + \frac{1}{2}\|e\|^2 \\ &\quad + \frac{1}{2}\frac{s_1^2}{\left(K^2 - s_1^2\right)^2}\rho_{\max}^2 + \frac{1}{2}\rho_{\max}^2\left\|\varepsilon_1^*\right\|^2 + \frac{3s_1^2}{2(K^2 - s_1^2)^2} \\ &\quad + \frac{1}{2}\rho_{\max}^2 s_2^2 + \frac{s_1^2}{2\left(K^2 - s_1^2\right)^2}\rho_{\max}^2 + \frac{1}{2}w_2^2 + \frac{1}{2}\left\|\bar{d}_1\right\|^2 \\ &\quad + \frac{s_1}{K^2 - s_1^2}\left(\rho\alpha_1 - \dot{y}_r + \theta_{g1}^T\varphi_1\left(\hat{\boldsymbol{\chi}}\right)\right) \\ &\quad + \frac{1}{\gamma_1}\tilde{\theta}_{g1}^T\left(\frac{\gamma_1 s_1}{K^2 - s_1^2}\varphi_1\left(\hat{\boldsymbol{\chi}}\right) - \dot{\theta}_{g1}\right) + \frac{1}{2}\|e\|^2\|P\|^2 \\ &\quad + \frac{\|K\|^2 s_1^2}{2\rho_{\min}} + \frac{\|K\|^2 y_r^2}{2\rho_{\min}^2} + \frac{\|K\|^4 y_r^2}{2\rho_{\min}^4} + \frac{1}{2}s_1^2. \end{aligned} \quad (40)$$

Therefore,

$$\begin{aligned} \dot{V}_1 &\le -\left(\lambda_{\min}(Q) - \frac{3}{2} - \frac{\|P\|^2}{2}\right)\|e\|^2 + M_2 \\ &\quad + \frac{s_1}{K^2 - s_1^2}\begin{pmatrix} \rho\alpha_1 + \frac{s_1}{K^2 - s_1^2}\rho_{\max}^2 + \frac{3s_1}{2(K^2 - s_1^2)} \\ -\dot{y}_r + \theta_{g1}^T\varphi_1\left(\hat{\boldsymbol{\chi}}\right) \end{pmatrix} \\ &\quad + \frac{1}{\gamma_1}\tilde{\theta}_{g1}^T\left(\frac{\gamma_1 s_1}{K^2 - s_1^2}\varphi_1\left(\hat{\boldsymbol{\chi}}\right) - \dot{\theta}_{g1}\right) + \frac{1}{2}\rho_{\max}^2 s_2^2 \\ &\quad + \frac{1}{2}w_2^2 + \frac{\|K\|^2 s_1^2}{2\rho_{\min}} + \frac{1}{2}s_1^2 \end{aligned} \quad (41)$$

where $M_2 = M_1 + (1/2)\rho_{\max}\|\varepsilon_1^*\|^2 + (1/2)\|\bar{d}_1\|^2 + [(\|K\|^2 y_r^2)/(2\rho_{\min}^2)] + [(\|K\|^4 y_r^2)/(2\rho_{\min}^4)]$.

Since control coefficient is unknown, the virtual control law described by Nussbaum function and adaptation laws are obtained as follows:

$$\alpha_1 = N(\eta)\begin{pmatrix} c_1 s_1 + \frac{s_1}{K^2-s_1^2}\rho_{\max}^2 + \frac{3s_1}{2(K^2-s_1^2)} - \dot{y}_r \\ +\theta_{g_1}^T\varphi_1(\hat{\chi}) + \frac{\|K\|^2(K^2-s_1^2)}{2\rho_{\min}}s_1 + \frac{s_1(K^2-s_1^2)}{2}\end{pmatrix} \quad (42)$$

$$\dot{\theta}_{g_1} = \frac{\gamma_1 s_1}{K^2 - s_1^2}\varphi_1(\hat{\chi}) - \sigma_1\theta_{g_1} \quad \theta_{g_1}(0) = 0 \quad (43)$$

$$\dot{\eta} = \frac{s_1}{d(K^2 - s_1^2)}\begin{pmatrix} c_1 s_1 + \frac{s_1}{K^2-s_1^2}\rho_{\max}^2 + \frac{3s_1}{2(K^2-s_1^2)} \\ -\dot{y}_r + \theta_{g_1}^T\varphi_1(\hat{\chi}) + \frac{\|K\|^2(K^2-s_1^2)}{2\rho_{\min}}s_1 \\ +\frac{s_1(K^2-s_1^2)}{2}\end{pmatrix} \quad (44)$$

where c_1, σ_1, and d are positive constants parameters to be designed.

By substituting (42)–(44) into (41), one can get

$$\dot{V}_1 \leq -\left(\lambda_{\min}\left(Q - \frac{3}{2} - \frac{\|P\|^2}{2}\right)\right)\|e\|^2 + M_2 - \frac{c_1 s_1^2}{K^2 - s_1^2} + \frac{1}{2}\rho_{\max}^2 s_2^2 + \frac{1}{2}w_2^2 + d(\rho N(\eta) + 1)\dot{\eta} + \frac{\sigma_1}{\gamma_1}\tilde{\theta}_{g_1}^T\theta_{g_1}. \quad (45)$$

It is noticeable that, α_1 must be passed through a first-order filter with dynamic z_2 and time constant of the filter is τ_2. This technique avoids the explosion of complexity in the backstepping technique

$$\tau_2\dot{z}_2 + z_2 = \alpha_1 \quad z_2(0) = \alpha_1(0). \quad (46)$$

By defining $w_2 = z_2 - \alpha_1$, the following relations are obtained:

$$\dot{z}_2 = -\frac{w_2}{\tau_2} \Rightarrow \dot{w}_2 = \dot{z}_2 - \dot{\alpha}_1 = -\frac{w_2}{\tau_2} + B_2(.) \quad (47)$$

where $\tau_2 > 0$ and $B_2(.)$ is a function of s_1, s_2, w_2, θ_{g_1}, y_r, $\dot{y}_r$, and $\ddot{y}_r$.

Step 2: Considering $s_2 = \hat{\chi}_2 - z_2$ and by using $\dot{\hat{\chi}}_2 = \hat{\chi}_3 - k_2\hat{\chi}_1 + \theta_2^T\varphi_2(\hat{\chi})$, the derivative of s_2 is

$$\dot{s}_2 = \dot{\hat{\chi}}_2 - \dot{z}_2 = \hat{\chi}_3 - k_2\hat{\chi}_1 + \theta_2^T\varphi_2(\hat{\chi}) - \dot{z}_2. \quad (48)$$

Consider the following Lyapunov function V_2:

$$V_2 = V_1 + \frac{1}{2}s_2^2 + \frac{1}{2}w_2^2 + \frac{1}{2\gamma_2}\tilde{\theta}_2^T\tilde{\theta}_2 \quad (49)$$

where $\gamma_2 > 0$ is a design parameter. By using (45), the derivative of V_2 along the time is obtained as follows:

$$\begin{aligned}\dot{V}_2 \leq &-\left(\lambda_{\min}(Q) - \frac{3}{2} - \frac{\|P\|^2}{2}\right)\|e\|^2 + M_2 - \frac{c_1 s_1^2}{K^2 - s_1^2} \\ &+ \frac{1}{2}\rho_{\max}^2 s_2^2 + \frac{1}{2}w_2^2 + d(\rho N(\eta) + 1)\dot{\eta} + \frac{\sigma_1}{\gamma_1}\tilde{\theta}_{g_1}^T\theta_{g_1} \\ &+ s_2\left(\underbrace{\hat{\chi}_3}_{\{5\}} - k_2\hat{\chi}_1 + \theta_2^T\varphi_2(\hat{\chi}) - \dot{z}_2\right) \\ &+ w_2\left(-\frac{w_2}{\tau_2} + B_2(.)\right) \\ &+ \frac{1}{\gamma_2}\tilde{\theta}_2^T\left(\gamma_2 s_2\varphi_2(\hat{\chi}) - \dot{\theta}_2\right) + \underbrace{\left|s_2\beta_2^*\right|}_{\{6\}}. \end{aligned} \quad (50)$$

In order to simplify (50), the following inequality is used:

$$|x| - x\tanh\left(\frac{x}{\varsigma}\right) \leq 0.2785\varsigma = \varsigma', \quad \varsigma > 0. \quad (51)$$

Therefore, in (50), by substituting $s_3 = \hat{\chi}_3 - z_3$, $w_3 = z_3 - \alpha_2$ in $\{5\}$ and using inequality (51) for managing term $\{6\}$, one can write

$$\begin{aligned}\dot{V}_2 \leq &-\left(\lambda_{\min}(Q) - \frac{3}{2} - \frac{\|P\|^2}{2}\right)\|e\|^2 + M_2 - \frac{c_1 s_1^2}{K^2 - s_1^2} \\ &+ d(\rho N(\eta) + 1)\dot{\eta} + \frac{\sigma_1}{\gamma_1}\tilde{\theta}_{g_1}^T\theta_{g_1} \\ &+ s_2\Bigg[s_3 + w_3 + \alpha_2 - k_2\hat{\chi}_1 + \theta_2^T\varphi_2(\hat{\chi}) - \dot{z}_2 \\ &\quad + \frac{1}{2}\rho_{\max}^2 s_2 + \beta_2^*\tanh\left(\frac{s_2\beta_2^*}{\varsigma}\right)\Bigg] + \varsigma' \\ &+ \frac{1}{2}w_2^2 - \frac{w_2^2}{\tau_2} + w_2 B_2(.) + \frac{1}{\gamma_2}\tilde{\theta}_2^T\left(\gamma_2 s_2\varphi_2(\hat{\chi}) - \dot{\theta}_2\right) \end{aligned} \quad (52)$$

where γ_2, ς, and β_2^* are the positive design constants.

In this step, the virtual control law and adaptation laws are obtained from the following relations:

$$\begin{aligned}\alpha_2 = &-c_2 s_2 + k_2\hat{\chi}_1 - \theta_2^T\varphi_2(\hat{\chi}) - \frac{1}{2}\rho_{\max}^2 s_2 \\ &- \beta_2^*\tanh\left(\frac{s_2\beta_2^*}{\varsigma}\right) + \dot{z}_2 \end{aligned} \quad (53)$$

$$\dot{\theta}_2 = \gamma_2 s_2\varphi_2(\hat{\chi}) - \sigma_2\theta_2 \quad \theta_2(0) = 0 \quad (54)$$

where c_2 and σ_2 are the positive design constants.

Substituting (53) and (54) into (52), yields

$$\begin{aligned}\dot{V}_2 \leq &-\left(\lambda_{\min}(Q) - \frac{3}{2} - \frac{\|P\|^2}{2}\right)\|e\|^2 + M_3 - \frac{c_1 s_1^2}{K^2 - s_1^2} \\ &+ d(\rho N(\eta) + 1)\dot{\eta} + \frac{\sigma_1}{\gamma_1}\tilde{\theta}_{g_1}^T\theta_{g_1} + \frac{\sigma_2}{\gamma_2}\tilde{\theta}_2^T\theta_2 + s_2(s_3 + w_3) \\ &+ w_2^2\left(\frac{1}{2} - \frac{1}{\tau_2}\right) + w_2 B_2(.) - c_2 s_2^2 \end{aligned} \quad (55)$$

where $M_3 = M_2 + \varsigma'$. It should be noted that α_2 must be passed through a first-order filter with dynamic z_3 and time constant τ_3

$$\tau_3\dot{z}_3 + z_3 = \alpha_2 \quad z_3(0) = \alpha_2(0). \quad (56)$$

Define $w_3 = z_3 - \alpha_2$, then we have

$$\dot{z}_3 = -\frac{w_3}{\tau_3} \Rightarrow \dot{w}_3 = \dot{z}_3 - \dot{\alpha}_2 = -\frac{w_3}{\tau_3} + B_3(.) \quad (57)$$

where $\tau_3 > 0$ and $B_3(.)$ is a function of variables s_1, s_2, s_3, w_2, w_3, θ_{g_1}, θ_2, y_r, $\dot{y}_r$, and $\ddot{y}_r$.

Step i: We consider $s_i = \hat{\chi}_i - z_i$, $3 \leq i \leq n-1$ and by using $\dot{\hat{\chi}}_i = \hat{\chi}_{i+1} - k_i\hat{\chi}_1 + \theta_i^T\varphi_i(\hat{\chi})$ the derivative of s_i is obtained as follows:

$$\dot{s}_i = \dot{\hat{\chi}}_i - \dot{z}_i = \hat{\chi}_{i+1} - k_i\hat{\chi}_1 + \theta_i^T\varphi_i(\hat{\chi}) - \dot{z}_i. \quad (58)$$

By using $s_{i+1} = \hat{\chi}_{i+1} - z_{i+1}$ and $w_{i+1} = z_{i+1} - \alpha_i$, (58) is written as

$$\begin{aligned}\dot{s}_i &= \hat{\chi}_{i+1} - k_i\hat{\chi}_1 + \theta_i^T\varphi_i(\hat{\chi}) - \dot{z}_i \\ &= s_{i+1} + w_{i+1} + \alpha_i - k_i\hat{\chi}_1 + \theta_i^T\varphi_i(\hat{\chi}) - \dot{z}_i. \end{aligned} \quad (59)$$

Analogously to the previous steps, α_i is passed through the first-order filter with time constant τ_{i+1} and dynamic z_{i+1}

$$\tau_{i+1}\dot{z}_{i+1} + z_{i+1} = \alpha_i \quad z_{i+1}(0) = \alpha_i(0). \tag{60}$$

By using $w_{i+1} = z_{i+1} - \alpha_i$, one can get the following equations:

$$\dot{z}_{i+1} = -\frac{w_{i+1}}{\tau_{i+1}} \Rightarrow \dot{w}_{i+1} = \dot{z}_{i+1} - \dot{\alpha}_i = -\frac{w_{i+1}}{\tau_{i+1}} + B_{i+1}(.) \tag{61}$$

where $\tau_{i+1} > 0$ and $B_{i+1}(.)$ is a function of variables $s_1, s_2, \ldots, s_{i+1}, w_2, w_3, \ldots, w_{i+1}, \theta_{g1}, \theta_2, \ldots, \theta_i, y_r, \dot{y}_r$, and $\ddot{y}_r$.

Define the following Lyapunov function candidate V_i:

$$V_i = V_{i-1} + \frac{1}{2}s_i^2 + \frac{1}{2}w_i^2 + \frac{1}{2\gamma_i}\tilde{\theta}_i^T\tilde{\theta}_i \tag{62}$$

where $\gamma_i > 0$ is a design parameter. The time derivative of V_i is obtained, and following the previous steps we have:

$$\begin{aligned}\dot{V}_i \le& -\left(\lambda_{\min}(Q) - \frac{3}{2} - \frac{\|P\|^2}{2}\right)\|e\|^2 + M_{i-1} \\ &- \frac{c_1 s_1^2}{K^2 - s_1^2} - \sum_{k=2}^{i-1} c_k s_k^2 + d(\rho N(\eta)+1)\dot{\eta} + \frac{\sigma_1}{\gamma_1}\tilde{\theta}_{g1}^T\theta_{g1} \\ &+ \sum_{k=2}^{i-1}\frac{\sigma_k}{\gamma_k}\tilde{\theta}_k^T\theta_k + \sum_{k=2}^{i-1} s_k s_{k+1} + \sum_{k=2}^{i-1} s_k w_{k+1} + \frac{1}{2}w_2^2 \\ &- \sum_{k=2}^{i-1}\left(\frac{1}{\tau_k}w_k^2 - w_k B_k(.)\right) \\ &+ s_i\big(s_{i+1} + w_{i+1} + \alpha_i - k_i\hat{\chi}_1 + \theta_i^T\varphi_i(\hat{\boldsymbol{\chi}}) - \dot{z}_i\big) \\ &+ |s_i\beta_i^*| - \frac{1}{\tau_i}w_i^2 + w_i B_i(.) + \frac{1}{\gamma_i}\tilde{\theta}_i^T\big(\gamma_i s_i\varphi_i(\hat{\chi}) - \dot{\theta}_i\big)\end{aligned} \tag{63}$$

where c_i, γ_i, σ_i, and β_i^*, $3 \le i \le n-1$, are the positive design constants.

In this step, the virtual control law and adaptation laws are obtained as follows:

$$\alpha_i = -c_i s_i + k_i\hat{\chi}_1 - \theta_i^T\varphi_i(\hat{\boldsymbol{\chi}}) - \beta_i^*\tanh\left(\frac{s_i\beta_i^*}{\varsigma}\right) + \dot{z}_i \tag{64}$$

$$\dot{\theta}_i = \gamma_i s_i\varphi_i(\hat{\boldsymbol{\chi}}) - \sigma_i\theta_i \quad \theta_i(0) = 0. \tag{65}$$

By substituting (64) and (65) into (63), it is obtained

$$\begin{aligned}\dot{V}_i \le& -\left(\lambda_{\min}(Q) - \frac{3}{2} - \frac{\|P\|^2}{2}\right)\|e\|^2 + M_i - \frac{c_1 s_1^2}{K^2 - s_1^2} \\ &- \sum_{k=2}^{i} c_k s_k^2 + d(\rho N(\eta)+1)\dot{\eta} + \frac{\sigma_1}{\gamma_1}\tilde{\theta}_{g1}^T\theta_{g1} \\ &+ \sum_{k=2}^{i}\frac{\sigma_k}{\gamma_k}\tilde{\theta}_k^T\theta_k + \sum_{k=2}^{i} s_k s_{k+1} + \sum_{k=2}^{i} s_k w_{k+1} + \frac{1}{2}w_2^2 \\ &- \sum_{k=2}^{i}\left(\frac{1}{\tau_k}w_k^2 - w_k B_k(.)\right)\end{aligned} \tag{66}$$

where $M_i = M_{i-1} + \varsigma'$ and $B_k(.)$ is a function of variables $s_1, s_2, \ldots, s_{k+1}, w_2, w_3, \ldots, w_{k+1}\ \theta_{g1}, \theta_2, \ldots, \theta_k, y_r, \dot{y}_r$, and $\ddot{y}_r$.

Step n: In this step, the actual control law u_d appears. This step is a final step. Consider $s_n = \hat{\chi}_n - z_n$, and since $\dot{\hat{\chi}}_n = -k_n\hat{\chi}_1 + \theta_n^T\varphi_n(\hat{\boldsymbol{\chi}}) + D(u)$, the derivative of s_i is

$$\dot{s}_n = \dot{\hat{\chi}}_n - \dot{z}_n = -k_n\hat{\chi}_1 + \theta_n^T\varphi_n(\hat{\boldsymbol{\chi}}) + D(u) - \dot{z}_n. \tag{67}$$

By using (6), $s_n = \hat{\chi}_n - z_n$ and $w_n = z_n - \alpha_{n-1}$, (67) is expressed as

$$\begin{aligned}\dot{s}_n =& \dot{\hat{\chi}}_n - \dot{z}_n = -k_n\hat{\chi}_1 + \theta_n^T\varphi_n(\hat{\boldsymbol{\chi}}) + D(u) - \dot{z}_n = -k_n\hat{\chi}_1 \\ &+ \theta_n^T\varphi_n(\hat{\boldsymbol{\chi}}) + u_d + \left(\tilde{d}_{mr} - \frac{u_d + \hat{d}_{mr}}{\hat{m}_r}\tilde{m}_r\right)\delta(t) \\ &+ \left(\tilde{d}_{ml} - \frac{u_d - \hat{d}_{ml}}{\hat{m}_l}\tilde{m}_l\right)(1-\delta(t)) + \varepsilon_d - \dot{z}_n.\end{aligned} \tag{68}$$

In this stage, the following ultimate Lyapunov function is considered:

$$\begin{aligned}V =& V_{n-1} + \frac{1}{2}s_n^2 + \frac{1}{2}w_n^2 + \frac{1}{2\gamma_n}\tilde{\theta}_n^T\tilde{\theta}_n + \frac{1}{2\zeta_1}\tilde{m}_l^2 \\ &+ \frac{1}{2\zeta_2}\tilde{m}_r^2 + \frac{1}{2\zeta_3}\tilde{d}_{ml}^2 + \frac{1}{2\zeta_4}\tilde{d}_{mr}^2\end{aligned} \tag{69}$$

where $\gamma_n > 0$, $\zeta_1 > 0$, $\zeta_2 > 0$, $\zeta_3 > 0$, and $\zeta_4 > 0$ are design parameters. The time derivative of V is obtained as

$$\begin{aligned}\dot{V} \le& -\left(\lambda_{\min}(Q) - \frac{3}{2} - \frac{\|P\|^2}{2}\right)\|e\|^2 + M_{n-1} - \frac{c_1 s_1^2}{K^2 - s_1^2} \\ &- \sum_{k=2}^{n-1} c_k s_k^2 + d(\rho N(\eta)+1)\dot{\eta} + \frac{\sigma_1}{\gamma_1}\tilde{\theta}_{g1}^T\theta_{g1} \\ &+ \sum_{k=2}^{n-1}\frac{\sigma_k}{\gamma_k}\tilde{\theta}_k^T\theta_k + \sum_{k=2}^{n-1} s_k s_{k+1} + \sum_{k=2}^{n-1} s_k w_{k+1} + \frac{1}{2}w_2^2 \\ &- \sum_{k=2}^{n-1}\left(\frac{1}{\tau_k}w_k^2 - w_k B_k(.)\right) \\ &+ s_n\Bigg[-k_n\hat{\chi}_1 + \theta_n^T\varphi_n(\hat{\boldsymbol{\chi}}) + u_d \\ &\quad + \left(\tilde{d}_{mr} - \frac{u_d + \hat{d}_{mr}}{\hat{m}_r}\tilde{m}_r\right)\delta(t) \\ &\quad + \left(\tilde{d}_{ml} - \frac{u_d - \hat{d}_{ml}}{\hat{m}_l}\tilde{m}_l\right)(1-\delta(t)) + \varepsilon_d - \dot{z}_n\Bigg] \\ &+ w_n\left(-\frac{w_n}{\tau_n} + B_n(.)\right) + \frac{1}{\gamma_n}\tilde{\theta}_n^T\big(\gamma_n s_n\varphi_n(\hat{\boldsymbol{\chi}}) - \dot{\theta}_n\big) + |s_i\beta_i^*| \\ &+ \frac{1}{\zeta_1}\tilde{m}_l\dot{\hat{m}}_l + \frac{1}{\zeta_2}\tilde{m}_r\dot{\hat{m}}_r + \frac{1}{\zeta_3}\tilde{d}_{ml}\dot{\hat{d}}_{ml} + \frac{1}{\zeta_4}\tilde{d}_{mr}\dot{\hat{d}}_{mr}.\end{aligned} \tag{70}$$

Therefore,

$$\begin{aligned}\dot{V} \le& -\left(\lambda_{\min}(Q) - \frac{3}{2} - \frac{\|P\|^2}{2}\right)\|e\|^2 + M_{n-1} - \frac{c_1 s_1^2}{K^2 - s_1^2} \\ &- \sum_{k=2}^{n-1} c_k s_k^2 + d(\rho N(\eta)+1)\dot{\eta} + \frac{\sigma_1}{\gamma_1}\tilde{\theta}_{g1}^T\theta_{g1} + \sum_{k=2}^{n-1}\frac{\sigma_k}{\gamma_k}\tilde{\theta}_k^T\theta_k\end{aligned}$$

$$
\begin{aligned}
&+\sum_{k=2}^{n-1} s_k s_{k+1} + \sum_{k=2}^{n-1} s_k w_{k+1} + \frac{1}{2} w_2^2 \\
&-\sum_{k=2}^{n-1}\left(\frac{1}{\tau_k} w_k^2 - w_k B_k(.)\right) \\
&+\frac{1}{\zeta_1}\tilde{m}_l\left(\dot{\hat{m}}_l - \zeta_1 \frac{u_d - \hat{d}_{ml}}{\hat{m}_l} s_n(1-\delta(t))\right) \\
&+\frac{1}{\zeta_2}\tilde{m}_r\left(\dot{\hat{m}}_r - \zeta_2 \frac{u_d + \hat{d}_{mr}}{\hat{m}_r} s_n \delta(t)\right) \\
&+\frac{1}{\zeta_3}\tilde{d}_{ml}\left(\dot{\hat{d}}_{ml} + \zeta_3 s_n (1-\delta(t))\right) \\
&+\frac{1}{\zeta_4}\tilde{d}_{mr}\left(\dot{\hat{d}}_{mr} + \zeta_4 s_n \delta(t)\right) \\
&+ s_n\Bigg[-k_n\hat{\chi}_1 + \theta_n^T \varphi_n(\hat{\chi}) - \dot{z}_n + u_d + \varepsilon_d \\
&\qquad + \beta_n^* \tanh\left(\frac{s_n \beta_n^*}{\varsigma}\right)\Bigg] + \varsigma' \\
&+ w_n\left(-\frac{w_n}{\tau_n} + B_n(.)\right) + \frac{1}{\gamma_n}\tilde{\theta}_n^T\left(\gamma_n s_n \varphi_n(\hat{\chi}) - \dot{\theta}_n\right) \qquad (71)
\end{aligned}
$$

where $\beta_n^* > 0$ is a constant design parameter.

In this step, the actual control law and corresponding adaptation laws are obtained as

$$
\begin{aligned}
u_d &= -c_n s_n - \frac{1}{2} s_n + k_n \hat{\chi}_1 - \theta_n^T \varphi_n(\hat{\chi}) + \dot{z}_n \\
&\quad - \beta_n^* \tanh\left(\frac{s_n \beta_n^*}{\varsigma}\right) \qquad (72)\\
\dot{\theta}_n &= \gamma_n s_n \varphi_n(\hat{\chi}) - \sigma_n \theta_n, \ \theta_2(0) = 0 \qquad (73)\\
\dot{\hat{d}}_{ml} &= -\zeta_3 s_n (1-\delta(t)) + a_3 \hat{d}_{ml} \qquad (74)\\
\dot{\hat{d}}_{mr} &= -\zeta_4 s_n \delta(t) + a_4 \hat{d}_{mr} \qquad (75)\\
\dot{\hat{m}}_l &= \zeta_1 \frac{u_d - \hat{d}_{ml}}{\hat{m}_l} s_n (1-\delta(t)) + a_1 \hat{m}_l \qquad (76)\\
\dot{\hat{m}}_r &= \zeta_2 \frac{u_d + \hat{d}_{mr}}{\hat{m}_r} s_n \delta(t) + a_2 \hat{m}_r \qquad (77)
\end{aligned}
$$

where c_n, σ_n, a_1, a_2, a_3, and a_4 are the positive design constants. By using the following inequality:

$$
s_n \varepsilon_d \le \frac{1}{2} s_n^2 + \frac{1}{2}\|\bar{\varepsilon}_d\|^2 \qquad (78)
$$

and substituting (72)–(78) into (71), yields

$$
\begin{aligned}
\dot{V} \le& -\left(\lambda_{\min}(Q) - \frac{3}{2} - \frac{\|P\|^2}{2}\right)\|e\|^2 - \frac{c_1 s_1^2}{K^2 - s_1^2} - \sum_{k=2}^{n} c_k s_k^2 \\
&+ d(\rho N(\eta)+1)\dot{\eta} + \frac{\sigma_1}{\gamma_1}\underbrace{\tilde{\theta}_{g_1}^T \theta_{g_1}}_{\{7\}} + \sum_{k=2}^{n}\frac{\sigma_k}{\gamma_k}\underbrace{\tilde{\theta}_k^T \theta_k}_{\{8\}} \\
&+ \sum_{k=2}^{n-1}\underbrace{s_k s_{k+1}}_{\{9\}} + \sum_{k=2}^{n-1}\underbrace{s_k w_{k+1}}_{\{10\}} + \frac{1}{2} w_2^2 + \frac{a_1}{\zeta_1}\underbrace{\tilde{m}_l \hat{m}_l}_{\{11\}} + \frac{a_2}{\zeta_2}\underbrace{\tilde{m}_r \hat{m}_r}_{\{12\}} \\
&+ \frac{a_3}{\zeta_3}\underbrace{\tilde{d}_{ml}\hat{d}_{ml}}_{\{13\}} + \frac{a_4}{\zeta_4}\underbrace{\tilde{d}_{mr}\hat{d}_{mr}}_{\{14\}} + M_n - \sum_{k=2}^{n}\left(\frac{1}{\tau_k} w_k^2 - \underbrace{w_k B_k(.)}_{\{15\}}\right) \qquad (79)
\end{aligned}
$$

where, $M_n = M_{n-1} + \varsigma' + (1/2)\|\bar{\varepsilon}_d\|^2$.

In addition, by using the Young's inequalities, the terms {7} and {8} in (79) can be written as follows:

$$
\tilde{\theta}_{g_1}^T \theta_{g_1} = \tilde{\theta}_{g_1}^T\left(\theta_{g_1}^* - \tilde{\theta}_{g_1}\right) \le -\frac{1}{2}\tilde{\theta}_{g_1}^T\tilde{\theta}_{g_1} + \frac{1}{2}\theta_{g_1}^{*^T}\theta_{g_1}^* \qquad (80)
$$

$$
\tilde{\theta}_k^T \theta_k \le -\frac{1}{2}\tilde{\theta}_k^T\tilde{\theta}_k + \frac{1}{2}\theta_k^{*^T}\theta_k^*, \ k = 2, \dots, n \qquad (81)
$$

and similarly, this is also true for terms {9} and {10} as

$$
\begin{aligned}
s_k s_{k+1} \le \frac{1}{2}s_k^2 + \frac{1}{2}s_{k+1}^2 \Rightarrow \sum_{k=2}^{n-1} s_k s_{k+1} &\le \sum_{k=2}^{n-1}\frac{1}{2}s_k^2 + \frac{1}{2}s_{k+1}^2 \\
&= \left(\frac{1}{2}s_2^2 + \dots + \frac{1}{2}s_{n-1}^2\right) \\
&\quad + \left(\frac{1}{2}s_3^2 + \dots + \frac{1}{2}s_{n-1}^2 + \frac{1}{2}s_n^2\right) \qquad (82)
\end{aligned}
$$

$$
\begin{aligned}
s_k w_{k+1} \le \frac{1}{2}s_k^2 + \frac{1}{2}w_{k+1}^2 \Rightarrow \sum_{k=2}^{n-1} s_k w_{k+1} &\le \sum_{k=2}^{n-1}\frac{1}{2}s_k^2 + \frac{1}{2}w_{k+1}^2 \\
&= \left(\frac{1}{2}s_2^2 + \dots + \frac{1}{2}s_{n-1}^2\right) \\
&\quad + \left(\frac{1}{2}w_3^2 + \dots + \frac{1}{2}w_{n-1}^2 + \frac{1}{2}w_n^2\right). \qquad (83)
\end{aligned}
$$

Using the same procedure, the terms {11}–{14} can be stated as follows:

$$
\tilde{m}_l \hat{m}_l = \tilde{m}_l\left(m_l^* - \tilde{m}_l\right) \le -\frac{1}{2}\tilde{m}_l^2 + \frac{1}{2}m_l^{*^2} \qquad (84)
$$

$$
\tilde{m}_r \hat{m}_r = \tilde{m}_r\left(m_r^* - \tilde{m}_r\right) \le -\frac{1}{2}\tilde{m}_r^2 + \frac{1}{2}m_r^{*^2} \qquad (85)
$$

$$
\tilde{d}_{ml}\hat{d}_{ml} = \tilde{d}_{ml}\left(d_{ml}^* - \tilde{d}_{ml}\right) \le -\frac{1}{2}\tilde{d}_{ml}^2 + \frac{1}{2}d_{ml}^{*^2} \qquad (86)
$$

$$
\tilde{d}_{mr}\hat{d}_{mr} = \tilde{d}_{mr}\left(d_{mr}^* - \tilde{d}_{mr}\right) \le -\frac{1}{2}\tilde{d}_{mr}^2 + \frac{1}{2}d_{mr}^{*^2}. \qquad (87)
$$

In addition for term {15} one can write

$$
w_k B_k(.) \le |B_k(.) w_k| \qquad (88)
$$

$$
|B_k(.) w_k| \le \frac{w_k^2 B_k^2(.)}{2\pi} + 2\pi, \ \pi > 0 \qquad (89)
$$

where $\pi > 0$ is a design parameter and substituting (80)–(89) into (79), one can write

$$
\begin{aligned}
\dot{V} \le& -(\lambda_{\min}(Q) - q)\|e\|^2 - \frac{c_1 s_1^2}{K^2 - s_1^2} - \sum_{k=2}^{n}\left(c_k - \frac{3}{2}\right)s_k^2 \\
&- \frac{1}{2}s_2^2 - s_n^2 + d(\rho N(\eta)+1)\dot{\eta} - \frac{\sigma_1}{2\gamma_1}\tilde{\theta}_{g_1}^T\tilde{\theta}_{g_1} + \frac{\sigma_1}{2\gamma_1}\theta_{g_1}^{*^T}\theta_{g_1}^* \\
&- \sum_{k=2}^{n}\frac{\sigma_k}{2\gamma_k}\tilde{\theta}_k^T\theta_k + \sum_{k=2}^{n}\frac{\sigma_k}{2\gamma_k}\theta_k^{*^T}\theta_k^* \\
&- \sum_{k=2}^{n}\left(\frac{1}{\tau_k} - \frac{1}{2} - \frac{B_k^2(.)}{2\pi}\right)w_k^2 + 2\pi(n-1) - \frac{a_1}{2\zeta_1}\tilde{m}_l^2 \\
&+ \frac{a_1}{2\zeta_1}m_l^{*^2} - \frac{a_2}{2\zeta_2}\tilde{m}_r^2 + \frac{a_2}{2\zeta_2}m_r^{*^2} - \frac{a_3}{2\zeta_3}\tilde{d}_{ml}^2 + \frac{a_3}{2\zeta_3}d_{ml}^{*^2} \\
&- \frac{a_4}{2\zeta_4}\tilde{d}_{mr}^2 + \frac{a_4}{2\zeta_4}d_{mr}^{*^2} + M_n \qquad (90)
\end{aligned}
$$

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where $q = (3/2) + (\|P\|^2/2)$. Let

$$A' = \left\{ \sum_{k=3}^{n} s_k^2 + \frac{1}{\gamma_1}\tilde{\theta}_{g_1}^T\tilde{\theta}_{g_1} + \sum_{k=2}^{n}\frac{1}{\gamma_k}\tilde{\theta}_k^T\theta_k + \sum_{k=2}^{n} w_k^2 + e^T P e \le 2p \right\}.$$

Therefore, B_k is a continuous function and there exists a positive constant H_k in the compact set A' such that $|B_k(.)| \le H_k$ on A'. Therefore, (90) can be rewritten in the following form:

$$\begin{aligned}\dot{V} \le& -(\lambda_{\min}(Q) - q)\|e\|^2 - \frac{c_1 s_1^2}{K^2 - s_1^2} - \sum_{k=2}^{n}\left(c_k - \frac{3}{2}\right)s_k^2 \\ &- \frac{1}{2}s_2^2 - s_n^2 + d(\rho N(\eta) + 1)\dot{\eta} - \frac{\sigma_1}{2\gamma_1}\tilde{\theta}_{g_1}^T\tilde{\theta}_{g_1} \\ &- \sum_{k=2}^{n}\frac{\sigma_k}{2\gamma_k}\tilde{\theta}_k^T\theta_k - \sum_{k=2}^{n}\left(\frac{1}{\tau_k} - \frac{1}{2} - \frac{H_k^2(.)}{2\pi}\right)w_k^2 - \frac{a_1}{2\zeta_1}\tilde{m}_l^2 \\ &- \frac{a_2}{2\zeta_2}\tilde{m}_r^2 - \frac{a_3}{2\zeta_3}\tilde{d}_{ml}^2 - \frac{a_4}{2\zeta_4}\tilde{d}_{mr}^2 + D \end{aligned} \quad (91)$$

where

$$\begin{aligned} D =& \frac{\sigma_1}{2\gamma_1}\theta_{g_1}^{*T}\theta_{g_1}^* + \sum_{k=2}^{n}\frac{\sigma_k}{2\gamma_k}\theta_k^{*T}\theta_k^* + \frac{a_1}{2\zeta_1}m_l^{*2} + \frac{a_2}{2\zeta_2}m_r^{*2} \\ &+ \frac{a_3}{2\zeta_3}d_{ml}^{*2} + \frac{a_4}{2\zeta_4}d_{mr}^{*2} + M_n + 2\pi(n-1). \end{aligned}$$

The parameters Q, c_k, τ_k, $k = 2, \ldots, n$ can be designed such that $\lambda_{\min}(Q) - q > 0$, $c_k - (3/2) > 0$, $(1/\tau_k) - (1/2) - (H_k^2(.)/2\pi) > 0$ for $k = 2, \ldots, n$. In addition, from (5), we have

$$-\frac{c_1 s_1^2}{K^2 - s_1^2} < -c_1 \log\left(\frac{s_1^2}{K^2 - s_1^2}\right). \quad (92)$$

Let

$$\begin{aligned} C = \min\Bigg\{& 2\frac{(\lambda_{\min}(Q) - q)}{\lambda_{\min}(P)}, 2c_1, 2\left(c_k - \frac{3}{2}\right), \frac{\sigma_1}{\gamma_1}, \frac{\sigma_k}{\gamma_k} \\ & 2\left(\frac{1}{\tau_k} - \frac{1}{2} - \frac{H_k^2(.)}{2\pi}\right), \frac{a_1}{\zeta_1}, \frac{a_2}{\zeta_2}, \frac{a_3}{\zeta_3}, \frac{a_4}{\zeta_4}\Bigg\} \\ & k = 2, \ldots, n. \end{aligned}$$

Consequently, (91) is expressed as follows:

$$\dot{V} \le -CV + d(\rho N(\eta) + 1)\dot{\eta} + D. \quad (93)$$

Therefore,

$$\frac{d}{dt}\left(Ve^{Ct}\right) = d(\rho N(\eta) + 1)\dot{\eta}e^{Ct} + De^{Ct}. \quad (94)$$

Integrating (94) from both sides yields

$$Ve^{Ct} = \int_0^t d(\rho N(\eta) + 1)\dot{\eta}e^{Ct}dt + \int_0^t De^{Ct}dt. \quad (95)$$

Following Lemma 1, $\int_0^t d(\rho N(\eta)+1)\dot{\eta}e^{C\tau}d\tau$ is bounded on $[0, t_f]$. Therefore, defining $D_{\max} = \max_{t\in[0,t_f]}\int_0^t d(\rho N(\eta) + 1)\dot{\eta}e^{C\tau}d\tau$, (94) can be expressed as

$$0 \le V(t) \le \left(D_{\max} + V(0) - \frac{D}{C}\right)e^{-Ct} + \frac{D}{C} \quad (96)$$

where (D/C), can be arbitrarily small [41].

Equation (96) shows that all the signals of the closed-loop system are SGUUB, and $|y - y_r| \le \sqrt{2V(0)}e^{-Ct} + \sqrt{2D'/C}$. This means that $|y - y_r|$ is bounded, but it does not converge to zero.

Based on the above formulation, we can finally state the following theorem.

Remark 2: The parameter selection recommendation for the DSC adaptive fuzzy backstepping design is given as follows. Select design parameters appropriately such that $\lambda_{\min}(Q) - q > 0$, $c_k - (3/2) > 0$, $(1/\tau_k) - (1/2) - (H_k^2(.)/2\pi) > 0$, then determine actual control law, virtual control law, and adaptive tuning law accordingly. These parameters guarantee the stability of the overall system. Moreover, from $|y - y_r| \le \sqrt{2V(0)}e^{-Ct} + \sqrt{2D'/C}$, we can conclude that by increasing C, or decreasing D' (which is related to design parameters), we can make the tracking error $y - y_r$ smaller. Nevertheless, it causes the magnitude of the control signal is larger. Subsequently, in practical control systems, a trade-off between control signal magnitude and the tracking error should be considered.

Remark 3: Since the proposed approach is in general form and contains a wide class of nonlinear systems, it can be applied to strict feedback form systems as well. In fact, if the system is represented in strict feedback form, the method presented in this paper can handle such systems with dead-zone, unavailability of the system states, unknown control direction, and output constraint.

Theorem 1: Consider an uncertain nonlinear nonstrict system (10) with unknown input dead-zone and disturbances. Under Assumptions 1–3, and using the control law (12) based on (72), and adaptation laws (74)–(72) for dead-zone, and adaptation laws given for each step, then the proposed adaptive fuzzy nonlinear controller based on observer dynamics (24), guarantees the closed-loop states are bounded and the tracking error is SGUUB.

Proof: Following each step of the design based on the proposed DSC method for the nonstrict system (10), the proof is straightforward and is concluded after step n of the design procedure. ■

V. Simulation Results

In this section, the effectiveness of the proposed method and is evaluated by a simulation example.

Consider the following nonstrict nonlinear system with unknown control direction:

$$\begin{cases} \dot{x}_1 = x_2 + f_1(x_1, x_2) + d_1(t) \\ \dot{x}_2 = \rho D(u) + f_2(x_1, x_2) + d_2(t) \\ y = x_1 \end{cases} \quad (97)$$

where $f_1(x_1, x_2) = 0.1(x_1 + x_2)$, $f_2(x_1, x_2) = x_1 x_2$, $d_1(t) = 0.1\sin(t)$, $d_2(t) = 0.1\cos\ (t)$, the signal reference is $y_r(t) = \sin\ (t)$. The parameters of dead-zone are $m_r = 2$, $m_l = 1.3$, $d_r = 0.4$, $d_l = -0.6$, and membership functions are chosen as

$$\mu_{F_i^l}(\hat{x}_i) = \exp\left[-\frac{(\hat{x}_i - 6 + 2l)}{2}\right], \ l = 1, 2, \ldots, 6. \quad (98)$$

Define fuzzy basis function as follows:

$$\varphi_{i,l}(\hat{x}_1, \hat{x}_2) = \frac{\mu_{F_i^l}(\hat{x}_1)\mu_{F_i^l}(\hat{x}_2)}{\sum_{l=1}^{5} \mu_{F_1^l}(\hat{x}_1)\mu_{F_2^l}(\hat{x}_2)}, \quad l = 1, 2, \ldots, 6 \; i = 1, 2 \tag{99}$$

and the FLS is

$$\hat{f}_i(\hat{x}_1, \hat{x}_2|\theta_i) = \theta_i^T \varphi_i(\hat{x}_1, \hat{x}_2), \quad i = 1, 2. \tag{100}$$

The virtual control law α_1, actual control law u_d, and adaption laws are determined as

$$\alpha_1 = N(\eta)\Bigg(c_1 s_1 + \frac{s_1}{K^2 - s_1^2}\rho_{\max}^2 + \frac{3s_1}{2(K^2 - s_1^2)} - \dot{y}_r + \theta_{g_1}^T \varphi_1(\hat{\chi}) + \frac{\|K\|^2(K^2 - s_1^2)}{2\rho_{\min}} s_1 + \frac{s_1(K^2 - s_1^2)}{2}\Bigg) \tag{101}$$

$$\dot{\theta}_{g_1} = \frac{\gamma_1 s_1}{K^2 - s_1^2}\varphi_1(\hat{\chi}) - \sigma_1\theta_{g_1}, \; \theta_{g_1}(0) = 0 \tag{102}$$

$$\dot{\eta} = \frac{s_1}{d(K^2 - s_1^2)} \times \Bigg(c_1 s_1 + \frac{s_1}{K^2 - s_1^2}\rho_{\max}^2 + \frac{3s_1}{2(K^2 - s_1^2)} - \dot{y}_r + \theta_{g_1}^T \varphi_1(\hat{\chi}) + \frac{\|K\|^2(K^2 - s_1^2)}{2\rho_{\min}} s_1 + \frac{s_1(K^2 - s_1^2)}{2}\Bigg) \tag{103}$$

$$u_d = -c_2 s_2 - \frac{1}{2}s_2 + k_2\hat{\chi}_1 - \theta_2^T\varphi_2(\hat{\chi}) + \dot{z}_2 - \beta_2^* \tanh\left(\frac{s_2\beta_2^*}{\varsigma}\right) \tag{104}$$

$$\dot{\theta}_2 = \gamma_2 s_2 \varphi_2(\hat{\chi}) - \sigma_2\theta_2, \; \theta_2(0) = 0 \tag{105}$$

$$\dot{\hat{m}}_l = -\zeta_1 \frac{u_d - \hat{d}_{ml}}{\hat{m}_l} s_2(1 - \delta(t)) - a_1\hat{m}_l \tag{106}$$

$$\dot{\hat{m}}_r = -\zeta_2 \frac{u_d + \hat{d}_{mr}}{\hat{m}_r} s_2\delta(t) - a_2\hat{m}_r \tag{107}$$

$$\dot{\hat{d}}_{ml} = \zeta_3 s_2(1 - \delta(t)) - a_3\hat{d}_{ml} \tag{108}$$

$$\dot{\hat{d}}_{mr} = \zeta_4 s_2\delta(t) - a_4\hat{d}_{mr}. \tag{109}$$

In addition, the design parameters are chosen as

$$k_1 = 0.1, \; k_2 = 0.6, \; c_1 = 20, \; c_2 = 5, \; d = 0.04, \; K = 2$$
$$\rho_{\min} = 1, \; \rho_{\max} = 3, \; \gamma_1 = 2 \; \gamma_2 = 2, \; \sigma_1 = 10, \; \sigma_2 = 10$$
$$\beta_n^* = 0.5, \; \varsigma = 5, \; \zeta_1 = 6, \; \zeta_2 = 10, \; \zeta_3 = 6, \; \zeta_4 = 2$$
$$a_1 = 2, \; a_2 = 1, \; a_3 = 3, \; a_4 = 2, \; \rho = \pm 2.$$

Besides, the initial conditions are selected as

$$(x_1(0), x_2(0)) = (0.2, -0.2)$$
$$\left(\hat{m}_r(0), \hat{m}_l(0), \hat{d}_{mr}(0), \hat{d}_{ml}(0)\right) = (1.5, 1.5, 1.5, 1.5)$$
$$\left(\hat{X}_1(0), \hat{X}_2(0)\right) = (0.1, 0.1)$$

and other initial values are chosen as zero.

The simulation results are carried out for two different gain signs to show the efficiency of the proposed approach. Figs. 1–3 are illustrated for $\rho = -2$ and Figs. 4 and 5 are illustrated for $\rho = 2$. As it is seen from Fig. 1, one can conclude that the output signal can track the reference trajectory with bounded error considering the boundedness of the output signal. In addition, the output is constrained by a predefined bound.

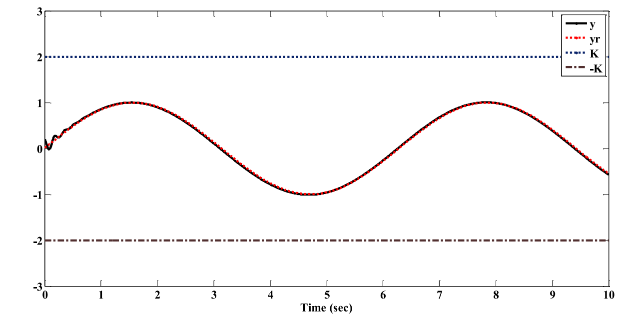

Fig. 1. Output y and the reference signal y_r for $\rho = -2$.

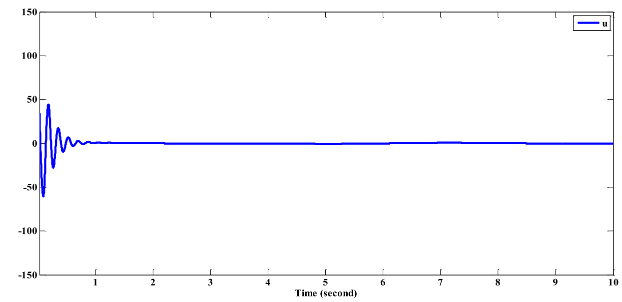

Fig. 2. Control input for $\rho = -2$.

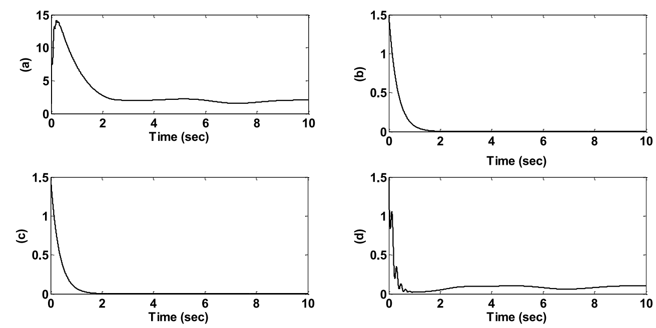

Fig. 3. Adaptive parameters for dead-zone nonlinearity: (a) $\hat{m}_r$, (b) $\hat{m}_l$, (c) $\hat{d}_{ml}$, and (d) $\hat{d}_{mr}$ for $\rho = -2$.

The control signal for this case is demonstrated in Fig. 2. It is obvious that this signal is bounded and applicable in practice.

The adaptive parameters of dead-zone nonlinearity are shown in Fig. 3. This figure show that these parameters are updated based on the proposed adaptive laws and vary during the simulations. In addition, these parameters are bounded and therefore, this makes the control signal bounded.

In addition, in order to show that in the proposed method it is not required to know *a priori* knowledge about control gain sign, the same simulations have been carried out for the proposed system when $\rho = 2$. From the simulation results, one can write even though the system is in this nonstrict feedback form, the system states may not be available

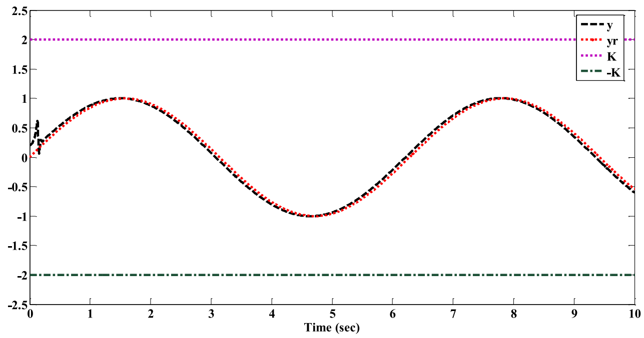

Fig. 4. Output y and the reference signal y_r for $\rho = 2$.

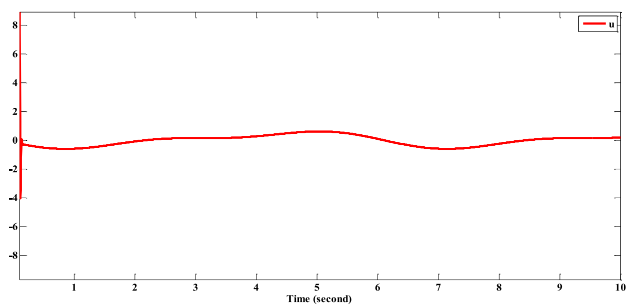

Fig. 5. Control input for $\rho = 2$.

for measurement, the system contains nonlinearities, unknown dead-zone, and unknown control direction, and the proposed observer-based fuzzy adaptive controller can guarantee the stability of the closed-loop system. In addition, as it is seen in Fig. 4, the system output tracks the desired trajectory with small tracking error. Besides, the system output does not violate the predefined bounded considered in the design procedure. In addition, it can be observed from Fig. 5 that the control signal is bounded and applicable for implementation. The proposed method can be implemented on a wide class of nonlinear systems in nonstrict feedback form. For example, it can be applied to practical systems, e.g., mass-spring-damper system [23] and marine surface vehicles [44].

VI. Conclusion

In this paper, we have developed a new method for controlling uncertain nonstrict nonlinear systems with known disturbances, unknown gain sign, and unknown dead-zone. In order to approximate unknown functions in the system, FLS has been utilized and based on the adaptive mechanism; the unknown functions were approximated effectively. In addition, to solve the problem of so-called "explosion of complexity" which exists in traditional back-stepping control design, the DSC methodology was used. This method incorporates a fuzzy logic observer to overcome the difficulty of accessibility to unmeasured states. It was shown that the proposed observer-based fuzzy adaptive controller can assure the boundedness of the closed-loop signals and ensures the tracking performance of the system in spite of having uncertainties, disturbances, and unknown control direction. Finally, a simulation example was demonstrated to show the efficiency of the proposed method.

It is interesting to apply the proposed method to practical examples in the future works. For example, the systems presented in [44] and [45] are two potential applications that could be good candidates for the proposed method. In addition, an extension of the proposed method to stochastic and multiagent nonlinear systems is considered as future work.

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